MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY.

VOL. LXXVIII. JANUARY 11, 1918. No. 3

Major P. A. MACMAHON, D.Sc., Sc.D., LL.D., F.R.S., PRESIDENT, in the Chair.

The Rev. Arthur Harold Young Baxter, M.A.Cantab., Tresillion, Mylor, near Falmouth, Cornwall;
Walter George Bell, 31 Baskerville Road, S.W. 18;
Capt. Charles John Philip Cave, R.E., M.A., J.P., F.R.Met.Soc., Ditcham Park Petersfield;
George Craig, F.I.C., 95 Bath Street, Glasgow;
John Crighton, B.Sc., War Signal Station, Beachy Head;
Henry Mason Garner, B.A., 97 Eltham Road, Lee, S.E. 12;
Alfred Holness, 13 and 14 Paternoster Row, E.C. 4;
Alfred George Maddock, Sub.-Lieut., R.N.V.R., F.R.Met.Soc., Meteorological Office, Royal Naval Air Service, 6 Brooklyn Terrace, Keyham, Devonport, and R.N. Air Station, Isle of Grain, Kent;
William George de Winton Mitchell, B.Sc., The Grammar School, Newbury, Berkshire, and 52 Craven Road, Newbury; and
The Rev. Roger Conyers Morrell, M.A., L.Th., F.R.G.S., F.Z.S., St. Thomas's Vicarage, Camden Town, N.W. 1,

were balloted for and duly elected Fellows of the Society.

The following candidate was proposed for election as a Fellow of the Society, the name of the proposer from personal knowledge being appended:—

Ronald Malcolm, Headley Grove, Headley, Epsom (proposed by the Rev. T. E. R. Phillips).

Thirty-two presents were announced as having been received since the last meeting, including, amongst others :—

A collection of Sir William Herschel's manuscripts, presented by Miss Francisca Herschel.

On the Photographic Magnitudes determined with the Vatican Astrographic Equatorial : Corrections depending on distance from the plate centre. By H. H. Turner, D.Sc., F.R.S., Savilian Professor.

1. To find the constant a in the adopted formula

$$\text{mag.} = a - 1{\cdot}02\sqrt{d}$$

used for the Vatican plates in Zone $+64°$, Miss E. F. Bellamy enters the magnitudes from the Helsingfors Catalogue on cards, adding the magnitudes calculated as above with a standard value of a and other particulars. It seemed desirable to add the distance r from plate centre, in reseau intervals, from a table of double entry in x and y; and the cards were then distributed in order of r, with subdivisions according to magnitude, measurer, etc. On discussing the results, one or two points of general interest emerged.

2. In the first instance, stars fainter than 8·0 Helsingfors were separated from those brighter. The mean residuals in units of 0·01 magnitude were as follows :—

Dist.	3	5	6	7	8	9	10	11	12	13	15	17
Bright	+16	0	+32	+59	+53	+17	+16	+43	+49	+50	+38	+10
Faint	−16	−18	−17	+ 6	+ 9	+ 5	+10	+ 8	− 7	−10	−16	−73
Diff.	+32	+18	+49	+53	+44	+12	+ 6	+35	+56	+60	+54	+83

It will be seen that the difference is sensibly smallest for a distance 9 or 10 reseau intervals, say 45′ from the plate centre. Further, that the bright ($<8{\cdot}0$) stars have a minimum at this point and the faint ($>8{\cdot}0$) stars a maximum. To see where the change occurs greater subdivision of magnitude is required, and the following groups were formed :—

Limits (Helsingfors)	<7·0	7·1–8·0	8·1–8·5	8·6–9·0	9·1–9·5
Estimated Mean	6·5	7·6	8·35	8·85	9·35
Corrected Mean (Harvard)	6·5	7·6	8·4	9·1	10·1

In the last line corrections have been applied to the Helsingfors magnitudes from *H. A.*, **23**, 184, assuming that they are practically on the D.M. scale. Further, the two measurers E and M were kept separate. The results are as in Table I.

TABLE I.

Vatican Mag.—Helsingfors Corrected in Units of 0·01 *Mag.*

Measurer E.

Mag. \ r=	3·	5·	6·	7·	8·	9·	10·	11·	12·	13·	15·	17·	Mean.
6·5	+24	+120	+134	+158	+ 97	+99	−24	+81	+92	+105	+50	+ 81	+74
7·6	+37	+ 18	+ 51	+ 32	+ 46	+23	+52	+62	+57	+ 41	+36	− 5	+43
8·4	+37	+ 3	+ 3	+ 35	+ 20	+17	+18	+11	+18	+ 8	− 5	−105	+10
9·1	− 4	− 38	− 26	+ 15	− 3	− 2	+11	+ 6	− 4	− 12	−29	− 65	− 9
10·1	−21	+ 1	− 27	+ 8	+ 3	− 3	+ 1	−12	−23	− 51	−24	− 64	−19

Measurer M.

6·5	+ 8	−117	− 82	+ 15	+138	−69	−28	−62	+39	+ 37	+28	...	−10
7·6	+11	+ 21	+ 24	+ 26	+ 34	+11	+29	+32	+25	+ 38	+40	...	+27
8·4	+11	− 47	0	+ 5	+ 26	+12	+32	+41	0	+ 16	+16	+ 13	+13
9·1	−31	− 13	− 5	+ 1	+ 24	+13	+25	+22	0	+ 20	−18	− 93	+ 2
10·1	−33	− 2	− 25	− 12	− 13	+ 5	− 7	−10	−11	− 10	−32	−103	−15

3. Looking first at the last lines, where stars are plentiful, we see that $r=9$ is a sharp maximum in M's results, and has a max. in its neighbourhood in E's results. Much the same is true of the line above. Taking these four lines together we get for their mean

r=	3	5	6	7	8	9	10	11	12	13	15	17
	−22	−13	−21	+3	+3	+3	+ 8	+ 2	− 9	−13	−26	−81

The best focus for the faint stars is thus in an annulus extending from $r=7$ to $r=11$, that is, $r=35'$ to $r=55'$. Outside this, the defect of focus rapidly increases; but the defect of focus near the centre is also clearly marked. We may now divide the plate into three portions, the central portion 0–6, the focal annulus 7–11, and the outside portion 12–15 (excluding the last figures under heading 17), and take the mean results from Table I. as in Table II.

TABLE II.

	E.				M.			
	Centre.	Focus.	Outside.	Diff.	Centre.	Focus.	Outside.	Diff.
6·5	+93	+82	+82	− 6	−64	− 1	+35	+16
7·6	+35	+43	+45	+ 3	+19	+26	+34	0
8·4	+14	+20	+ 7	+10	−12	+23	+11	+23
9·1	−23	+ 5	−15	+24	−16	+17	+ 1	+25
10·1	−16	− 1	−33	+24	−20	− 7	−18	+12

4. The columns headed "Diff." give the excess of the column "focus" over the mean of the other two. It seems clear that the loss of magnitude by defective focus persists from the bottom up to the second line. For magnitude 7·6 it is small, but is not

negative in either set of measures. Whether above this there is an actual reversal cannot be definitely stated, as the E and M results are at variance; and if we go back to Table I., we see that the figures for the brightest stars are affected with large accidental errors, owing to the paucity of observations.

5. There would be no difficulty in explaining an actual reversal, of which clearer indication was expected after the results given in § 2 had been obtained. The explanation has indeed been already given as regards the outer portions of a plate in a communication from Greenwich with a title from which that of the present paper was deliberately copied (*M.N.*, 73, 519):—

"An interesting point is that the imperfect definition at the edges of the plate affects bright and faint stars in opposite ways: the diameter of a bright star is made larger by being slightly out of focus, whereas a faint star may either be lost entirely, or will leave a fainter impression, which will be judged to have a smaller diameter."

6. Reference to Table IV. in that paper shows that the central portions might have been explicitly included with the edges in the above statement, as they are implicitly on the evidence of Table IV. The evidence there collected depends on simple counts of stars without any comparison with visual magnitudes. Owing to the accidental errors introduced by these visual magnitudes (which bring additional sources of error from which the Greenwich counts are free), and possibly also to the superior delicacy of the test of simple counts (to which attention is called in the Greenwich paper), the Greenwich results show greater accuracy than those given above. But the check by a method independent of simple counts is of some interest, for the reality of the whole phenomenon was directly challenged by the late M. Loewy in a passage already quoted (*M.N.*, 68, 397), on the ground that it was not confirmed by direct experiments made for the purpose of testing it.

7. In the last column of Table I. are given the mean errors for different magnitudes. If we omit the results for the brightest stars, it will be seen that the other four numbers are in good accordance for the two observers, but agree in showing that the adopted constant 1·02 in the formula

$$\text{mag.} = a - 1{\cdot}02\sqrt{d}$$

is not correct. The true interval, $10{\cdot}10 - 7{\cdot}60 = 2{\cdot}50$, is shown by the Vatican reductions as $9{\cdot}93 - 7{\cdot}95 = 1{\cdot}98$. This points to the use of too small a factor in the ratio 2·50 to 1·98, or 1·25 to 1. Hence the adopted factor 1·02 should apparently be 1·27. Reference to Table I. will show how easily the wrong factor may be obtained unless the bright stars are excluded. The measures of M show the bright stars (6·5) in good accord with the faintest (10·1); in other words, the factor 1·02 was deduced from the extremes of this series. But there were at least two other reasons for the adoption of the smaller factor at a time when it was necessary to adopt some first approximation. Firstly, the "true"

interval is only 2·50 if we adopt the Harvard corrections to the Helsingfors magnitudes. The latter as they stand would give an interval $9{\cdot}4 - 7{\cdot}6 = 1{\cdot}8$, corresponding to a *reduced* coefficient 0·92 instead of an increased coefficient 1·25. Secondly, our Oxford experience had led us to think that the tendency had been to adopt too high a value of this coefficient in some zones. The question of a revised value is receiving consideration. It would be easy to adopt a value from counts of the stars compared with the scale of Chapman and Melotte, but that doubts have recently been cast on the finality of this scale.

The Compressibility of Dwarf Stars and Planets.

By Harold Jeffreys, M.A., D.Sc.

If a particular piece of a substance has volume v and density ρ at pressure p, the modulus of incompressibility is defined as

$$k = -v\frac{\partial p}{\partial v} = \rho\frac{\partial p}{\partial \rho}$$

where the temperature is supposed kept constant. Regarding k as a function of p, we have at once

$$\log \rho = \mathrm{A} + \int \frac{dp}{k},$$

where A is a constant. Thus if k were always finite, or even proportional to p, ρ could be made to increase indefinitely by increasing p. This is impossible, for ρ can never be greater than the density when the atoms are as closely packed as possible. It is therefore theoretically evident that k must tend to infinity with p, so that k/p tends to infinity, and it is experimentally known that k actually does increase rapidly with it.

Let us consider to what extent this increase in incompressibility will affect the vibrations of a large star. The condition that any part of a body can be treated as incompressible is that in it the velocity of a compression wave, gravity being neglected, shall be great compared with that of a gravity wave, the body being regarded as incompressible. Now the former velocity is c, given by

$$c^2 = \frac{\partial p}{\partial \rho} = \frac{k}{\rho}.$$

Hence $c/p^{\frac{1}{2}}$ tends to infinity with p. The period of a gravity wave is always of order $\rho_0^{-\frac{1}{2}}$, where ρ_0 is the mean density, and is not greatly affected by changes in the size of the body. Thus its velocity c' is of the same order as a, the radius of gyration of the body. Now p, at any depth comparable with a, is of order a^2. Thus c/c' tends to infinity with a. Hence, finally, any body can be treated as incompressible throughout most of its bulk, provided only that it is large enough.

Looking at the matter in another way, the condition required is that c^2 shall be large compared with c'^2. Now c'^2 is of the same order as ga, where g is the value of gravity at distance a from the centre. Thus we require that k shall be large compared with $g\rho a$, which is incidentally of the same order as the pressure at the centre.

In the case of the earth k is of order 10^{12} c.g.s. at the surface, while $g\rho a$ may reach about 5×10^{12}. It follows that k must increase considerably as we approach the centre, but it is not possible to say without further evidence that it is ever great in comparison with $g\rho a$; thus compressibility may be important in the earth.

Considering a larger body, such as the sun, suppose that the condition is satisfied at all depths greater than h, and try to find the condition that h may be a small fraction of a. In the case of the most compressible substance, a perfect gas, k is equal to p, and when the gas is much compressed it is considerably greater than p. Now let the depth h be measured from some point denoted by the suffix 0, where the density is, say, 0·3. Such a point will not be very far below the visible surface. Then assuming a surface temperature of 6000° and an average molecular weight of 60, we see that the pressure p_0 is of order 2×10^9 dynes per sq. cm. Assuming further that k increases with p so as always to be greater than $k_0(p/p_0)^{1+\gamma}$, we have the condition at depth h

$$k_0\left(\frac{g\rho_0 h}{p_0}\right)^{1+\gamma} > g\rho a.$$

Now remembering that k_0 is greater than p_0, $\rho_0 = 0{\cdot}3$, $\rho < 1{\cdot}4$, we at once find that h/a is of order $(5 \times 10^{-6\gamma})^{1/(1+\gamma)}$, and if γ is as large as 0·2, it follows that h/a is small. Similarly it can be shown that this will hold for Jupiter if γ is as large as 0·4.

Van der Waal's law of density can be expressed in the form

$$p = \frac{\kappa\rho}{\rho_0 - \rho} - a\rho^2;$$

leading to

$$k = \frac{\rho\rho_0\kappa}{(\rho_0 - \rho)^2} - 2a\rho^2.$$

Thus when p is large and $\rho_0 - \rho$ small, k is practically proportional to p^{-2}, making $\gamma = 1$. Hence if a dwarf star or a planet comparable in size with Jupiter is as difficult to compress as an imperfect gas, which is highly probable, h/a will be small. When this holds, in problems dealing with compression waves, gravity may be neglected, save in a comparatively thin surface layer; similarly in problems dealing with phenomena of long period such bodies may be treated as incompressible except near the surface.

It is known that, so far as the velocities of waves in the earth have been determined, the ratio of the rigidity to the incompressibility varies very little with the depth. If the same were true in larger bodies, it would follow that most of their bulk might be treated as perfectly rigid in dealing with phenomena of long period.

Note on the Light Curves of Short-period Variables.

By Rev. T. E. R. Phillips, M.A.

1. In their paper on RZ Cephei, presented at the December meeting of the Society, Professor Plummer and Mr. C. Martin show that the light curves of this and seven other short-period variables observed at Dunsink exhibit characteristics in striking agreement with those of stars in the second of the two groups into which the long-period variables are divided (*B. A. A. Journal*, **27**, 5 *et seq.*).

The great difference of period renders such an accordance worth further investigation; and with a view to obtaining additional evidence bearing on the problem, I have submitted to harmonic analysis the light curves of the eighteen short-period variables in Professor Pickering's Class IV., which are published in *Harvard Annals*, **46**.

The adopted expression for the light curves is

$$M + k_1 \cos(\theta - \phi_1) + k_2 \cos(2\theta - \phi_2) + k_3 \cos(3\theta - \phi_3).$$

In each case the twenty-four ordinates given have been utilised, and for mutual comparison the curves have all been reduced to the maximum of the first harmonic as origin, so that in every case $\phi_1 = 180°$.

2. The result of the analysis will be seen in the accompanying tables, the outstanding fact which emerges being that apparently the short-period variables are also divided into two groups, though the evidence for one of them is at present less strong than that for the other.

Table I. contains a list of five stars which may be compared with those of Group I. in the long-period variables (*B. A. A. Journal*, **27**, 6). It will be remembered that in that group ϕ_2 has a considerable range, but ϕ_3 in the mean remains nearly constant in the neighbourhood of 200°. Now the stars in Table I. show these general characteristics very well, though the mean value of ϕ_3 (168°) is rather small. Since, however, the stars are so few in number, and the mean coefficient of ϕ_3 is less than three-hundredths of a magnitude, we cannot attach much weight to the difference, which may disappear as other stars are added to the list.

TABLE I.

Star.	Period.	Range.	Mean Magnitude.	Coefficients.			Phases.		
				k_1.	k_2.	k_3.	ϕ_1.	ϕ_2.	ϕ_3.
	d	m		m	m	m	°	°	°
ζ Geminorum	10·15	0·56	4·02	0·27	0·02	0·01	180	230	128
S Antliæ	0·32	0·44	6·53	0·20	0·02	0·01	,,	295	186
RS Sagittarii	2·42	0·42	6·03	0·14	0·11	0·05	,,	342	162
β Lyræ	12·91	0·74	3·58	0·15	0·27	0·03	,,	14	182
W Geminorum	7·74	0·77	7·07	0·32	0·04	0·03	,,	32	184

Three light curves of *S Antliæ* — determined by three different processes—are given in *Harvard Annals*, **46**. These have been combined, and the curve analysed is the mean of the three.

The star *β Lyræ* should perhaps be excluded from the list, as there is reason to believe that its light changes depend on some kind of eclipse. If so, it is probably radically different in character from the other stars, but other interpretations seem possible. The light curve is interesting. The fourth harmonic is

$$0^{m}\cdot05\cos(4\theta - 330^{\circ}),$$

so that k_2 and k_4 are considerably larger than k_1 and k_3 respectively, while at the adopted origin the second and fourth harmonics are roughly in opposite phase to the first and third.

3. Turning now to the stars in Table II. it will be seen that they fall well into line both with those given in the list of Messrs. Martin and Plummer and with the long-period variables of Group II. (*B. A. A. Journal*, **27**, 7). Messrs. Martin and Plummer remark on the small range of ϕ_2 in the Dunsink stars, but the range is considerably extended in those observed at Harvard, and is approximately equal to that shown by the corresponding group in the long-period variables.

TABLE II.

Star.	Period.	Range.	Mean Magnitude.	Coefficients.			Phases.		
				k_1.	k_2.	k_3.	ϕ_1.	ϕ_2.	ϕ_3.
	d	m		m	m	m	°	°	°
S Sagittæ	8·38	0·63	5·71	0·31	0·06	0·05	180	56	300
W Sagittarii	7·59	0·85	4·78	0·32	0·14	0·06	,,	76	29
U Sagittarii	6·74	0·79	6·86	0·34	0·10	0·02	,,	77	269
RV Scorpii	6·06	0·63	7·07	0·29	0·08	0·05	,,	87	73
T Vulpeculæ	4·44	0·60	5·86	0·31	0·08	0·04	,,	93	23
X Cygni	16·39	1·01	6·53	0·46	0·12	0·06	,,	98	80
Y Sagittarii	5·77	0·74	5·79	0·35	0·06	0·05	,,	103	92
δ Cephei	5·37	0·88	4·15	0·44	0·08	0·03	,,	108	14
T Monocerotis	27·01	1·12	6·37	0·46	0·14	0·07	,,	108	56
U Aquilæ	7·02	0·75	4·11	0·33	0·16	0·04	,,	119	93
η Aquilæ	7·18	0·79	4·11	0·34	0·15	0·03	,,	124	100
Y Ophiuchi	17·12	0·46	6·34	0·22	0·03	...	,,	141	...
X Sagittarii	7·01	0·67	4·73	0·30	0·03	0·02	,,	163	173

Two light curves—determined by different methods—are given in *Harvard Annals* for both δ *Cephei* and T *Monocerotis*. The mean of the two curves in each case is here analysed.

The third harmonic of *Y Ophiuchi* is practically insensible, and it is therefore omitted from the table. The general form of the curve, however, shows that the star belongs to this group.

4. With the evidence now available, it seems that there is a marked tendency of variable stars of all known periods (including the sun *) to exhibit light curves conforming to one or other of two types with distinct characteristics. That designated Group II. is obviously the one specially favoured, but as material accumulates it will probably be found that the number of stars showing the features of Group I. is not inconsiderable, even among the variables of short period.

5. In view of Mr. Shapley's theory of pulsations in a single body, adopted by Messrs. Martin and Plummer, it would seem desirable for some Observatory suitably equipped to undertake the search for variations in the radial velocity of some of the long-period variables synchronising with the light changes of the star.

Headley:
1918 *January* 10.

Observations of Nova Geminorum (2) 1912.

By J. M. Baldwin, M.A., D.Sc.

The following estimates of magnitude of Nova Geminorum (2) were made by me, using the finder of the 8-inch equatorial (aperture $2\frac{1}{4}$ inches, magnification 10), except where the estimated magnitude is 5·0 or brighter, in which cases the naked eye was employed. The magnitudes throughout are on the Potsdam scale. The delay in forwarding these observations, originally prepared for publication in 1912 May, has been due to circumstances beyond my control.

G.M.T. 1912	h	m	Mag.	Comp. Stars.	Remarks.
March 13	23	38	(3·6)	...	Rough estimate under very unfavourable conditions.
14	22	7	4·2	*a*, *b*	Thin cloud about.
	23	4	4·2		
15	22	47	5·5	*d*, *e*	Hazy low down.
	23	6	5·7		
	23	23	5·7		
16	21	40	5·6	*d*, *e*	
	22	41	5·4		
	23	24	5·4		
17	22	22	5·4	*d*, *e*	Hazy.

* The sun-spot curve was included in Professor Turner's series, *M.N.*, 67, 5.

G.M.T. 1912	h	m	Mag.	Comp. Stars.	Remarks.
March 18	22	23	5·7	*d*, *e*	Slightly hazy.
19	21	47	5·7	*d*, *e*	
	22	36	5·6		
20	21	21	5·8	*e*	
21	...		...	...	Thick variable haze; no comparison, but probably Nova is brighter than last night.
22	21	54	5·3	*c*, *d*, *e*	
23	22	44	4·8	*b*, *c*	
24	21	30	5·0	*b*, *c*	
28	21	39	5·9	*e*	Difficult to get a comparison on account of cloud.
31	21	6	5·9	*e*, *f*	
April 1	21	22	6·2	*f*	
3	22	10	6·2	*e*, *f*	
5	22	32	6·4	*f*, *g*	
10	21	8	6·6	*f*, *g*	
11	21	8	6·8	*g*, *h*	
12	21	35	6·8	*g*, *h*	
14	21	7	6·8	*g*, *h*	Clear in region; much thin cloud about.
16	21	13	6·9	*g*, *h*	
28	20	45	7·0	*g*, *h*	
May 2	20	35	7·1	*g*, *h*	

Comparison Stars.

		P.D.N°.	Mag.		P.D.N°.	Mag.
a	θ Gemin.	4229	3·84	*e*	4200	5·84
b	ρ Gemin.	4563	4·40	*f*	4271	6·18
c	τ Gemin.	4401	4·59	*g*	4317	6·80
d	28 Gemin.	4160	5·56	*h*	4243	7·22

Melbourne Observatory:
1917 *November.*

Micrometrical Measures of Double Stars (12*th Series*), *and New Double Stars.* By Rev. T. E. Espin, M.A.

In the following list the measures have been made with the 24-in. Calver during 1917:—

Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	1917.
	h m	° ′	°	″			
Espin 151	0 3·7	40 5	194·4	6·49	8·5 10·4	3	·800
Espin 152	8·1	39 40	103·7	7·72	8·5 9·8	2	·865
Gr. 34	12·7	43 27	195·9	19·59	8·0 11·0	2	·890
Espin 222	20·6	38 14	152·1	6·42	8·6 11·3	2	·917
Espin 223	41·5	38 15	262·7	4·16	9·5 9·6	2	·906
Σ 64	45·7	40 40	272·2	3·35	8·7 8·9	2	·865
h 1064	57·3	40 48	3·7	19·68	7·2 12·0	3	·873
J. 875	1 2·7	40 3	337·1	2·96	10·6 10·6	2	·870
A.G.	9·0	40 30	45·8	6·43	8·5 8·9	2	·011
h 1087	33·4	38 30	73·8	11·95	8·9 9·6	2	·918
Σ 141	34·2	38 28	301·3	1·79	8·1 8·3	2	·916
Σ 215	2 2·9	40 19	60·2	19·84	8·2 9·0	2	·800
A.G. 32	3·0	40 22	98·4	20·99	8·9 9·2	2	·800
Σ 222	4·8	38 34	36·2	16·71	7·0 7·5	1	·977
Espin 229	5·2	37 37	41·7	1·81	9·5 10·5	2	·934
Σ 245	11·8	39 50	291·8	11·06	8·0 8·2	2	·890
Espin 164	11·9	40 24	326·7	8·00	8·8 13·0	2	·105
σ Andromedæ	16·6	41 2	1·6	56·31	6·5 11·0	2	·011
A.G. 42	28·0	39 51	143·5	6·41	8·7 9·5	2	·890
J. 881	32·7	37 3	24·3	3·06	9·3 9·7	2	·932
A.G. 49	36·1	38 11	342·2	14·37	8·6 9·2	2	·947
Espin 231	36·6	37 59	82·4	4·29	8·9 10·1	2	·947
A.G. 51	41·8	37 8	272·2	4·66	9·1 9·3	2	·934
Espin 232	46·6	39 29	188·0	2·28	8·9 10·1	2	·916
B.D. +40°·762	3 22·3	40 18	310·9	11·32	8·3 9·5	3	·614
ρ 76	27·3	40 27	34·3	4·20	9·4 10·6	2	·830
Espin 166	35·1	39 23	1·5	4·61	8·6 12·2	2	·924
J. 17	5 58·0	43 3	150·2	2·73	9·1 9·2	3	·153
J. 910	6 21·3	43 8	336·0	1·54	9·1 9·2	2	·061 AB
			30·1	42·61	C= 9·4	2	·061 AC
A. 2449	21·9	42 43	243·1	1·79	9·1 9·2	2	·196
58 Aurigæ	43·7	41 54	101·7	40·71	5·0 9·3	3	·251

Name.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	1917.
	h m	° ′	°	″			
Σ 1279	6 43·4	39 57	89·3	1·41	8·8 8·9	2	·217
Σ 1344	9 17·2	39 34	101·0	3·41	8·5 8·8	2	·257
Espin 297	18·8	39 11	41·9	3·71	8·8 12·2	2	·257
Espin 298	19·4	39 2	308·3	7·28	8·7 11·2	4	·272 AB
			168·5	3·68	10·1 11·3	4	·272 CD
A.G.	25·4	38 56	264·0	17·41	9·4 9·5	2	·286
h 2499	27·9	38 53	319·3	9·66	9·2 10·5	2	·286
Σ 1374	35·2	39 25	285·7	3·10	7·5 8·1	2	·257
h 3322	10 7·6	38 16	130·0	7·70	9·5 9·8	2	·311
Σ 1420	12·6	39 36	324·6	2·16	8·5 9·1	2	·257
+38°·2175	38·6	38 12	233·9	8·85	9·2 9·5	2	·331
h 2555	57·7	39 7	46·6	11·24	10·0 11·5	3	·296
Espin 306	11 33·0	39 18	342·6	6·89	8·2 10·5	2	·311
h 510	45·1	38 15	252·5	24·62	9·7 10·1	2	·339
Espin 307	12 0·1	39 24	4·4	5·37	8·1 13·5	3	·294
Σ 1601	1·0	39 22	311·6	1·87	8·2 8·6	2	·311
Σ 2030	16 9·3	41 2	241·2	5·45	8·0 10·5	1	·531
h 2802	53·7	39 16	125·2	11·87	9·5 13·0	1	·550
A.G. 225	18 36·5	40 29	357·2	6·41	9·2 9·3	4	·724 AB
			239·8	10·44	C= 13·5	4	·724 AC (new)
Webb	41·8	44 51	49·8	10·99	8·8 9·2	1	·769
ρ 50	42·0	39 58	279·5	4·28	9·5 10·6	2	·765
Espin 81	19 23·1	39 56	218·3	11·48	8·2 11·5	3	·817
Espin 82	23·4	40 7	178·6	2·16	8·5 10·0	3	·759
J. 800	33·0	41 25	171·7	1·79	9·3 10·5	2	·685
Scheiner 673	40·3	39 39	47·5	10·50	9·5 11·5	2	·844 AB
			261·7	17·61	C= 12·5	2	·844 AC
Scheiner 682	44·1	39 42	64·7	4·20	9·5 10·1	2	·844 AB
Scheiner 683			22·7	19·88	C= 11·7	2	·844 AC
J. 787	20 18·9	40 27	157·8	2·59	9·6 12·0	2	·690 AB
			107·0	8·96	C= 12·5	2	·750 AC
			290·8	15·65	D= 12·0	2	·818 AD
De. 22	21·9	39 46	143·9	2·78	8·0 8·3	3	·868 AB
			94·6	17·36	C= 14·0	2	·865 AC
Espin 245	26·5	39 47	154·9	3·66	9·5 9·6	3	·859
Scheiner 1161	29·0	39 40	273·6	8·64	9·4 9·9	2	·870
Scheiner 1222	38·2	39 32	260·5	22·34	9·5 9·5	2	·898
Scheiner 1241	41·9	38 55	307·7	6·71	10·0 10·1	2	·909

Name.	R.A. 1900.		Decl.		P.	D.	Mags.		Nts.	1917.	
	h	m	°	′	°	″					
Scheiner 1267	20	46·1	39	36	310·8	2·89	10·5	10·8	4	·907	
Scheiner 1308		50·8	38	37	94·3	5·95	9·5	10·8	2	·917	
Scheiner 1363		56·3	39	49	38·6	8·97	8·8	9·3	2	·870	
Scheiner 1369		57·4	39	25	309·1	5·16	10·5	10·9	3	·913	
A.G.	21	4·4	40	30	104·9	5·47	8·9	9·0	2	·759	
Espin 255		7·1	40	7	32·7	4·36	9·3	11·2	2	·870	
Espin 256		9·0	40	7	280·1	4·24	9·5	9·5	2	·870	
Scheiner 1484		12·6	39	35	5·7	4·12	9·4	11·8	2	·909	
Scheiner 1503		15·7	39	23	274·0	9·89	9·8	10·4	2	·917	
Scheiner 1506		16·0	39	32	87·0	4·40	9·9	10·7	4	·900	
Scheiner 1535		41·8	39	15	229·0	8·13	10·6	11·2	4	·890	
Espin 261		48·2	39	52	164·2	4·86	9·5	9·6	2	·802	
h 1703		50·0	39	25	80·6	8·68	9·1	10·7	2	·882	
Scheiner 1545		52·9	38	29	326·1	5·28	9·6	9·9	2	·898	
Scheiner 1555	22	0·0	37	35	177·8	7·20	9·5	9·7	3	·953	
Scheiner 1556		0·3	38	25	246·6	10·27	9·4	10·2	2	·898	
Σ 2866		5·0	40	10	53·9	8·96	8·7	9·5	2	·724	
Scheiner 1562		12·1	37	29	141·9	19·68	9·0	10·6	2	·943	
Weisse 38		19·7	40	24	52·6	6·85	8·6	9·0	2	·805	
h 1788		30·5	41	3	295·6	3·16	8·7	8·9	2	·882	
Espin 262		32·4	37	9	163·0	5·35	9·3	10·8	2	·974	
Espin 263		32·1	40	32	241·5	8·84	9·3	13·9	4	·781	
Espin 264		33·0	40	30	0·0	8·17	9·1	12·2	4	·781	
β 176		38·0	38	46	41·2	2·27	9·1	9·2	2	·898	
Σ 2942		39·6	38	56	273·3	2·91	7·0	9·2	4	·912	AB
					237·9	9·84	C=	10·7	2	·924	AC
A.G. 287		41·0	39	30	193·1	14·09	8·5	9·0	2	·802	
Espin 266		48·8	39	48	83·5	13·32	8·1	11·2	3	·800	
h 1832		53·1	38	8	72·9	11·08	9·4	10·8	2	·940	
Hn. 55		55·0	39	44	190·0	1·83	8·7	9·5	3	·852	
Σ 2992	23	8·3	39	27	283·2	14·32	7·8	9·0	2	·882	
h 1882		22·9	38	52	322·3	13·24	8·9	13·2	2	·927	
OΣ 501		35·1	37	6	161·2	14·84	7·0	9·0	2	·974	
Σ 3042		46·8	37	21	85·5	4·98	8·1	8·2	2	·927	
Σ 3043		47·8	38	8	250·1	15·69	8·7	9·0	2	·916	
A.G. 296		50·7	37	57	53·9	5·72	9·0	9·1	2	·916	
A.G. 297		52·1	37	18	312·0	1·87	9·0	9·2	2	·974	
Espin 221		55·7	36	13	232·2	15·28	8·8	9·7	1	9·77	

Notes.

Gr. 34.—A nearer and fainter *comes* than the one which is moving with Gr. 34. A single setting made the angle of the 10 mag. at 56°·4. It is being measured by Mr. Franks and so was not measured with the 24-inch. The new *comes* is almost at right angles to the P.M. of Gr. 34, and measures next year will determine whether it is physically connected with the large star.

A.G., $1^h\ 9^m{\cdot}0$; A.G., $9^h\ 25^m$; A.G., $21^h\ 4^m{\cdot}4$.—None of these are in the β's General Catalogue.

Σ 222, $2^h\ 4^m{\cdot}8$. Milburn, P. 37°·0. D. 16″·64. 3 nt. 1918 ·008.

h 2449, $9^h\ 27^m{\cdot}9$, *h* 2555, $10^h\ 57^m{\cdot}7$.—There are no other measures of these stars.

Webb, $18^h\ 41^m{\cdot}8$.—Taken from Webb's MS. His observations: "1875 July 24, Mags. 10, 10·5 : 8″," bluish. Aug. 24, "10·5, 11 : P. 30°." Full measures of this pair are given in *M.N.*, **74**, 245, when it was measured in the course of the zone work as:

B.D. +44°·2985. P. 50°·1. D. 10″·80. Mags. 8·6, 9·0. 3 nt. 1913 ·754.

J. 787, $20^h\ 18^m{\cdot}9$.—The two *comites* were added with the 17¼-inch reflector by Mr. Milburn.

Scheiner 1484, $21^h\ 12^m{\cdot}6$. Also J. 1152.

Scheiner 1506; 1895·6. P. 93°·4. D. 4″·08. This is the only case among Scheiner's stars where there is large difference in angle.

Espin 263.—This is B.D. +40°·4859 *not* +40°·4860.

Σ 2942, $22^h\ 39^m{\cdot}6$.—The angles on the four nights were: 271°·7, 273°·7, 274°·0, 273°·7. On account of the great difference of these from the earlier observation, the angle on the last night was measured with the greatest care. The stars were beautifully defined, and when allowed to run along the wire, it was obvious that the angle could not possibly be 280°, or even 275°. The colours are fine orange and contrasted blue. It may at last be moving.

A.G. 297. Milburn, P. 313°·1. 1 nt. 1918 ·019.

Espin 221, $23^h\ 55^m{\cdot}7$. Milburn, P. 237°·4. D. 14″·83. 2 nt. 1918 ·019.

New Double Stars.

No.	B.D.		R.A. 1900.		Decl.		P.	D.	Mags.		Nts.	Date. 1917.	
	°		h	m	°	′	°	″					
1601	+39	9	0	3·4	+39	28	109·9	5·85	9·0	14·0	4	·871	
1602	40	60		15·5	40	16	85·6	7·12	9·6	9·9	2	·744	
1603	40	80		20·9	40	40	322·4	2·72	12·5	13·5	3	·754	BC
							55·5	28·07	A=	9·5	3	·754	AB
1604	40	134		33·9	40	17	158·4	2·58	9·5	13·0	5	·856	
1605	40	174		45·3	41	3	277·2	5·31	9·2	10·7	3	·794	
1606	41	155		48·0	41	21	58·3	1·71	9·4	11·2	4	·390	
1607	39	320	1	18·6	40	12	214·9	3·37	9·4	12·2	3	·821	
1608	37	315		28·9	38	10	42·4	1·20	9·5	9·7	3	·917	
1609	39	392		38·4	40	8	189·1	1·85	9·4	9·5	2	·877	
1610	...		2	3·5	40	25	287·4	2·77	11·0	11·1	3	·791	
1611	39	563		27·0	39	52	108·5	1·19	9·3	9·5	3	·898	
1612	40	583		38·0	40	40	204·1	3·14	9·3	9·6	3	·357	
1613	40	612		44·1	40	53	16·1	6·90	8·7	9·1	3	·102	
1614	40	658		56·9	40	32	128·8	2·51	9·4	11·1	2	·098	

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	°	h m	° ′	°	″			1917.
1615	40 670	3 0·9	40 25	208·9	1·73	9·2 9·4	2	·105
1616	40 911	4 7·4	40 47	233·0	6·29	8·5 9·8	2	·154
1617	40 984	26·2	40 29	212·0	2·89	9·3 10·8	2	·925
1618	41 948	37·8	41 58	337·0	3·29	9·5 11·3	3	·102
1619	41 949	38·6	41 46	353·0	4·89	9·6 10·0	2	·112
1620	42 1144	56·1	42 15	308·7	3·20	12·7 13·2	2	·195 BC
				106·9	6·54	D= 13·6	2	·195 BD
				8·2	13·95	A= 8·5	2	·195 AB
1621	42 1153	57·5	42 24	188·5	4·18	9·3 12·0	3	·194
1622	42 1200	5 5·4	42 51	247·7	5·50	9·3 9·5	1	·935
1623	42 1208	6·8	42 23	2·0	5·33	9·5 11·0	1	·935
1624	42 1211	7·3	42 49	47·9	2·79	9·5 9·8	1	·935
1625	41 1209	24·3	41 47	262·1	3·25	9·5 10·5	1	·197
1626	42 1393	39·7	42 48	2·1	7·52	9·5 11·9	2	·134
1627	42 1414	43·3	42 20	303·9	1·89	9·3 11·0	2	·193
1628	42 1491	6 1·7	42 43	286·3	3·15	9·2 10·1	2	·195
1629	43 1477	4·4	43 7	105·0	1·33	9·1 9·7	2	·171
1630	42 1562	23·9	42 57	91·4	4·95	9·7 9·8	4	·217
1631	41 1583	57·2	41 46	231·6	7·43	9·5 9·7	2	·134
1632	41 1584	58·3	41 36	227·6	3·87	9·4 12·0	2	·171 AB
				77·4	10·99	C= 11·6	3	·180 AC
1633	41 1585	58·8	41 43	27·5	2·81	9·5 9·7	2	·171
1634	...	7 11·3	40 17	86·9	4·01	9·5 9·6	2	·235
1635	40 1967	50·7	40 34	274·6	7·39	9·1 10·4	2	·134
1636	40 2016	8 10·3	40 6	265·7	5·27	9·5 12·2	2	·195
1637	39 2186	55·8	39 5	110·6	3·98	9·1 12·7	2	·257
1638	39 2285	9 47·4	39 7	71·3	2·96	9·5 9·8	4	·291
1639	38 2128	10 14·8	37 49	131·3	2·10	9·6 9·8	2	·331
1640	38 2188	47·5	38 44	228·7	2·83	9·5 9·6	3	·264
1641	39 2407	57·8	38 55	345·3	5·31	9·4 13·7	3	·277
1642	38 2212	11 4·1	38 38	44·5	1·28	9·5 9·7	2	·339
1643	39 2446	24·9	39 7	83·2	3·66	9·5 13·7	2	·249
1644	...	46·0	39 2	326·3	2·24	10·0 10·1	2	·249
1645	40 2960	15 57·6	39 52	213·3	3·14	9·5 9·6	3	·529
1646	40 3028	16 28·7	40 27	119·2	4·79	9·4 12·0	2	·543
1647	40 3048	37·1	40 32	19·4	3·32	9·5 10·7	3	·529
1648	41 2963	17 59·7	41 9	4·4	3·58	9·5 10·8	2	·535
1649	40 3292	18 4·6	40 1	331·4	6·79	9·5 13·0	2	·680 AB
				244·8	19·80	C= 12·5	2	·680 AC
1650	40 3309	8·8	40 13	142·8	1·62	9·4 10·3	4	·750

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	°	h m	° ′	°	″			1917.
1651	41 3021	18 12·6	41 5	358·7	2·87	9·6 10·7	3	·661
1652	40 3359	20·8	40 7	148·2	6·49	9·5 13·0	2	·709
1653	40 3375	23·7	40 27	177·8	7·50	9·5 9·7	2	·700
1654	40 3415	31·0	40 35	253·1	3·05	9·5 12·2	4	·680
1655	40 3437	34·5	40 9	27·8	2·06	9·4 10·2	3	·688
1656	41 3136	43·0	41 12	249·3	4·87	9·6 14·2	4	·750
1657	40 3524	52·1	40 44	355·8	3·78	9·4 10·0	3	·688
1658	39 3653	19 5·5	39 54	107·2	1·20	9·5 11·0	3	·799
1659	40 3605	5·8	40 48	252·2	6·03	9·5 12·3	3	·731
1660	41 3277	11·4	41 15	72·9	5·10	9·3 12·5	3	·670
1661	41 3328	19·5	41 8	146·4	3·81	9·5 12·0	2	·680
1662	40 3701	20·5	40 9	71·6	2·93	9·4 11·2	2	·770
1663	41 3350	23·7	41 17	228·8	5·85	8·4 12·7	3	·661 AB
				300·6	24·17	C= 11·8	3	·661 AC
1664	...	24·4	40 41	98·7	1·83	9·8 10·2	2	·739
1665	41 3406	31·7	41 11	72·1	5·96	9·3 12·2	2	·700
1666	40 3809	34·2	40 8	317·0	4·87	9·5 12·2	4	·812
1667	40 3812	34·4	40 17	99·4	4·03	8·5 12·0	2	·792
1668	39 3856	35·2	40 6	142·2	2·61	9·5 10·5	2	·810
1669	40 3845	37·9	40 44	153·0	1·45	9·3 9·4	3	·777
1670	41 3464	39·7	41 10	97·6	3·56	9·5 12·0	2	·711
1671	41 3500	46·1	41 9	64·4	4·20	9·4 9·5	2	·711
1672	...	50·4	41 7	111·4	2·10	9·5 11·1	2	·690
1673	40 3947	52·1	40 33	187·5	3·61	8·6 11·7	3	·788
1674	40 4102	20 14·5	41 3	22·6	4·58	9·1 9·2	2	·770
1675	...	16·0	41 14	144·3	2·54	9·6 10·2	3	·746
1676	...	18·5	40 23	286·3	2·20	9·8 10·2	3	·813
1677	40 4146	19·6	40 26	268·3	6·79	9·4 11·6	2	·690
1678	41 3737	21·3	42 3	181·1	7·28	9·0 14·0	2	·707
1679	40 4212	28·0	40 52	216·7	4·51	9·4 12·0	3	·801
1680	...	29·1	39 35	21·1	2·31	9·7 10·4	2	·870
1681	40 4317	44·3	41 3	144·0	2·16	9·2 9·4	2	·690
1682	40 4320	44·9	40 19	149·8	2·79	9·5 9·7	3	·829
1683	40 4341	47·9	40 42	202·5	4·99	9·5 9·6	3	·746
1684	40 4351	50·5	40 48	274·2	7·51	9·2 12·7	2	·703
1685	40 4355	50·9	40 19	74·5	1·73	12·7 13·5	3	·681 BC
				75·0	21·57	A= 9·5	2	·674 AB
1686	40 4358	51·8	40 49	192·9	5·99	8·5 12·0	3	·708
1687	40 4399	21 0·7	40 33	43·2	6·81	9·1 9·4	2	·670
1688	40 4466	12·5	41 4	145·5	5·20	9·5 9·6	2	·739

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.		Nts.	Date.
	°	h m	° ′	°	″				1917.
1689	40 4467	21 12·6	40 44	137·7	3·22	9·5	9·7	3	·758
1690	...	12·6	40 21	105·9	3·75	9·5	10·5	2	·802
1691	40 4499	17·8	40 28	161·2	6·84	8·5	13·0	3	·799
1692	40 4514	21·1	40 55	322·4	3·49	9·4	11·2	3	·762
1693	39 4663	41·7	40 1	41·8	6·93	8·5	12·5	3	·874
1694	40 4681	53·3	40 14	70·6	1·65	9·4	10·1	3	·732
1695	39 4746	22 0·7	39 40	40·3	1·12	9·5	10·0	3	·781
1696	...	25·8	40 29	141·1	2·32	10·8	11·2	4	·751
1697	...	29·6	40 20	338·7	2·76	10·0	10·7	2	·792
1698	39 4888	31·1	39 22	31·2	2·22	9·5	11·2	2	·865
1699	40 4886	38·1	40 39	106·3	1·53	9·5	9·6	2	·739
1700	...	41·0	39 33	236·4	4·14	9·5	11·5	3	·818
1701	39 4961	49·4	39 28	66·8	7·14	9·5	9·6	3	·812
1702	40 4948	51·8	40 22	261·4	2·59	9·2	10·3	3	·687
1703	40 5007	23 6·0	41 5	115·2	3·93	9·5	11·0	2	·765
1704	40 5040	13·1	40 18	336·1	2·05	9·3	10·9	4	·747
1705	Espin 268	41·9	39 59	274·2	0·82	9·1	9·3	4	·866 AB
				263·1	4·09	C = 10·7		5	·798 $\frac{AB}{2}$ C

Notes.

No. 1610.—In field with B.D. +40°·446 at P. 121°·9.
1612.—*f* star of two in Argelander.
1614.—Also a 10 mag. at P. 57°·6. D. 37″.
1615.—13′ from Algol at P. 229°·1.
1619.—A 10·5 mag. in same direction.
1622.—Milburn, P. 252°·8. D. 5″·55. 1 nt. 18·00.
1631.—S. of two. Declination of the two stars in Argelander has apparently been reversed.
1672.—Forms a distant *comes* to B.D. +40°·3931 at P. 49°·3.
1680.—Forms a distant *comes* to B.D. +39°·4228 at P. 6°·2.
1696.—Forms a distant *comes* to B.D. +40°·4835 at P. 344°·5.
1702.—N. star of wide pair.
1705.—Mr. Jonckheere has kindly measured this with the 28-inch at Greenwich, with the following results:—

AB	P. 277°·5	D. 1″·01	Mags. 9·1, 10·3	3 nt.	1917	·850
AC	P. 267°·8	D. 4″·30	C = 11·2	3 nt.	1917	·850

The following stars have been detected by Mr. W. Milburn with the 17¼-inch, and have been measured with the 24-inch.

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.		Nts.	Date.
	°	h m	° ′	°	″				1917.
1	...	0 21·5	40 18	237·5	6·22	11·9	12·3	3	·821
2	...	24·3	39 37	234·2	3·57	9·8	9·9	3	·799
...	38 99	37·7	38 39	350·9	11·28	9·5	9·7	2	·889
3	...	41·7	38 28	16·	7·60	9·6	9·8	3	·911

No.	B.D.	R.A. 1900.	Decl.	P.	D.	Mags.	Nts.	Date.
	°	h m	° ′	°	″			1917.
4	38 114	0 42·9	38 51	97·6	6·54	9·5 10·6	2	·932 AB
				164·1	18·20	C= 10·0	1	·934 AC
5	39 230	55·2	39 57	342·8	6·62	9·5 9·7	2	·932
6	...	1 30·9	39 36	58·7	7·32	10·0 10·0	2	·835
7	38 324	36·1	39 4	316·9	3·99	9·6 10·6	3	·874
8	38 331	37·2	38 46	98·6	6·39	9·1 9·6	2	·870
9	40 388	46·3	40 55	322·8	5·85	8·7 9·3	2	·783
10	...	57·4	38 50	51·5	3·35	10·0 10·1	2	·877
11	39 458	58·2	39 37	127·3	2·14	9·5 9·7	3	·828
12	38 418	2 2·9	39 5	256·1	5·70	8·6 9·7	3	·898
13	39 600	34·6	39 48	132·2	7·12	9·5 9·7	2	·870
14	...	3 0·4	39 29	7·6	5·02	10·2 10·5	3	·947
15	...	0·7	39 44	230·1	2·33	11·0 11·4	3	·899
...	39 817	28·7	39 28	310·7	9·55	9·4 10·5	3	·888
16	38 802	39·0	38 50	316·2	6·56	8·8 11·7	3	·931
17	39 903	53·1	39 52	31·1	7·93	9·5 11·2	4	·941
18	...	4 13·1	40 14	151·1	3·16	9·7 9·9	2	·956
19	39 987	18·7	39 12	220·8	3·00	9·2 11·0	1	·977
20	40 4146	20 19·6	40 26	268·3	6·79	9·4 11·6	2	·690
21	...	20·7	39 41	333·0	5·64	11·7 12·0	3	·878
22	...	21·7	39 43	225·2	5·06	10·3 11·6	3	·868
...	...	22·8	40 9	95·0	8·80	9·5 9·7	2	·750
23	40 4284	38·5	40 19	42·7	7·76	8·6 10·5	3	·847
24	...	39·2	40 14	171·5	5·76	9·5 9·7	2	·684
25	40 4298	41·0	41 9	205·0	5·85	9·2 9·4	3	·693
26	40 4550	21 26·3	40 18	69·5	5·23	9·0 11·7	2	·781
27	39 4599	30·6	39 57	249·5	6·00	12·4 14·0	4	·762 CD
				281·2	18·89	A= 9·5	2	·743 AC
				323·5	17·76	B= 12·0	2	·743 AB
28	...	32·3	39 35	246·7	4·82	9·5 11·0	2	·865
29	40 4599	35·6	40 23	340·6	6·50	9·5 11·0	2	·798
30	...	37·9	40 31	229·2	7·67	10·0 11·7	3	·784
31	39 4749	22 1·3	39 38	309·1	7·47	9·5 9·7	2	·770
...	...	8·0	39 22	292·0	8·37	10·5 11·5	2	·865
32	...	28·1	39 16	354·6	5·93	10·1 10·2	3	·847
33	38 4894	49·9	38 14	298·6	5·73	9·4 10·8	3	·953
...	38 4947	23 6·7	38 50	109·8	10·60	9·5 10·7	2	·839
...	39 5056	14·0	39 42	139·9	8·79	8·8 11·0	3	·884
34	39 5164	42·8	39 30	327·9	7·99	9·5 10·9	3	·874
35	...	44·6	38 24	223·5	5·28	9·7 10·7	2	·953

A Missing Star.

The star

	h	m	s	°	′	
B.D. +38°.344	1	37	44·3	+38	57·6	Mag. 9·4 (1855·0)

was found to be invisible on 1917 December 8 by Mr. Milburn, observing with the 17¼-inch. It was looked for by me on two nights in December with the 24-inch, and no star found in this place. There is no star here on the Harvard map of the sky.

Occultations of Stars by the Moon observed at the Royal Observatory, Greenwich, in the year 1917.

(*Communicated by the Astronomer Royal.*)

Date and G.M.T. of Observation.				Star and Phase of Occultation.	Magnitude of Star.	Observer.	Aperture. inch.	Power.
	d	h	m	s					
Feb.	7	11	19	15·32	π Leonis D *	4·9	H	10	120
,,	12	21	10·32	,, R	...	H	10	120
Mar.	29	7	34	9·02	5 Geminorum D	5·9	H	10	120
,,	10	2	33·94	8 ,, D	6·1	H	10	225
,,	10	33	52·54	9 ,, D	6·2	H	10	225
,,	11	5	12·25	8 ,, R *	6·1	H	10	225
,,	11	24	15·64	9 ,, R *	6·2	H	10	225
,,	11	32	37·98	10 ,, D (*a*)	6·6	H	10	225
Apr.	27	8	52	7·98	B.D. +20° 1866 D (*b*)	7·8	AC	2¾	40
May	25	9	39	43·46	ζ^1 Cancri D	4·7	H	10	225
,,	9	39	45·07	,, D	...	HF	2¾	40
,,	9	39	52·84	ζ^2 Cancri D	6·1	H	10	225

The telescopes used are the 10-inch guiding telescope of the Astrographic Equatorial, and a portable telescope, aperture 2¾ inches, in the Altazimuth dome.

The asterisk indicates that the phenomenon occurred at the Moon's Bright Limb.

Notes.

(*a*) Some cloud, but probably the actual occultation was seen.

(*b*) Star faint.

The star 412 B Tauri did not disappear on April 25, its least distance from the limb being estimated at 5″. Observer AC.

The initials H, AC, HF are those of Mr. Hollis, Mr. Crommelin, and Mr. Furner.

Royal Observatory, Greenwich:
1918 *January* 10.

The Meteoric Shower of January. By W. F. Denning and Mrs Fiammetta Wilson.

This display was observed on the evening of January 3. At Bristol there was a slight fog, sufficiently dense to obscure stars below the third magnitude, except near the zenith. A watch was maintained during a portion of the time from 5.50 to 9.50 p.m., and 11 meteors of the January shower were recorded in the northern sky, not far from the radiant. They were rather bright, slow, or very slow, and were directed from a radiant $233^\circ + 59\frac{1}{2}^\circ$ near ι Draconis. This point is about 8° north of the normal position of the radiant, the discordance being a striking feature of the display.

At Totteridge observations were maintained from 5.35 to 9.5 and 9.40 to 10.45, and, in a good sky, 66 meteors were observed, and 44 recorded. The flights of what were presumed to be Quadrantids were slower than the usual meteors from this shower, and came from a radiant far north of the point observed in past years. The difference indeed was so marked that the observer was inclined to regard the shower as probably a new one. The meteors generally, however, had a consistency of direction from a radiant at $233^\circ + 58^\circ$, and no doubt formed a veritable return of the old January shower, but with a decided change in the point of radiation.

From a great number of determinations of the radiant in past years the mean position was at $231^\circ + 51^\circ$. Taking a mean of the recent observations at Bristol and Totteridge, the point this year was at $233^\circ + 58\frac{3}{4}^\circ$, or nearly 8° north of the former position.

Some observations at Bristol in 1916 January 3 and 1917 January 3 indicated a radiant far north of the usual place, but the data at the time were regarded as insufficient.

There has been a difference of opinion as to the most suitable name for this January shower the radiant of which lies in the region between Hercules, Draco, Boötes, and Ursa Major, and in that portion of the sky known as Quadrans Muralis (Bode). If, however, the radiant is maintained in future years at or near the place it occupied at the recent return of the shower, it is certain that the only suitable name for the meteors will be Draconids.

Good displays of this shower were recorded in 1825, 1835, 1838, 1839, 1840, 1848, 1864, 1879, 1880, 1892, 1897, 1900, 1906, and 1918. There are, however, considerable difficulties in the way of determining the years of actual maxima. Clouds or moonlight effectively veil the stream in certain years, and in some others its chief activity, which is of brief duration, occurs in the day-time. An apparently rich display of the meteors may therefore be induced in a large measure, not so much by a denser swarm of the objects as by more favourable conditions attending their observation.

The above years of maxima, if entitled to much weight, would seem to indicate a period of about $13\frac{1}{4}$ years in the periodical

returns of the best showers, for, taking these intervals from 1825, we get the following years :—

1825	1878
$1838\frac{1}{4}$	$1891\frac{1}{4}$
$1851\frac{1}{2}$	$1904\frac{1}{2}$
$1864\frac{3}{4}$	$1917\frac{3}{4}$

It must be confessed, however, that the evidence of periodicity is very slender, and that it remains for future observers to decide it, and whether or not specially abundant showers recur in about 1931 and 1944.

It is curious that during the recent shower of this year a very small proportion of the meteors seen on January 3 *did* radiate from the old position $231^\circ + 51^\circ$, but there was a marked difference in their motion, as they were decidedly swifter than the majority of meteors directed from the new radiant at $233^\circ + 58\frac{3}{4}^\circ$.

Several accordances have already been found amongst the records sent in by various observers. The radiants of these doubly observed meteors are as follows :—

$232^\circ + 55\frac{1}{2}^\circ$	$235^\circ + 59^\circ$
$231 + 60$	$232 + 61$
$230 + 60$	$232 + 60$
$233 + 55$	

1918 *January* 8.

On the Magnitudes of the Cape Photographic Durchmusterung.
By J. Halm.

The magnitudes published in the C.P.D. Catalogues are based on a provisional photometric system derived from the visual estimates of the magnitudes of stars observed in the southern B.D. and in the Argentine Catalogues and Zones. It is known, however, that below the 9th the magnitudes of these reference catalogues show systematic errors which must have considerably affected the scale of the C.P.D. magnitudes.

Numerous photometric observations were recently made at this Observatory for the purpose of standardising, on a correct photometric scale, the C.P.D. magnitudes within a zone centred on the parallel 45° south declination, with the ultimate view of deriving photographic magnitudes of the stars contained in the Cape Astrographic Zones.

The first step in this direction was the establishment of standard photographic magnitudes for a sequence of stars in the vicinity of the South Pole. In two previous papers (*M.N.*, **74** and **75**) the method by which these magnitudes were determined has been described in detail. In Table A the provisional

photographic magnitudes of 77 south polar stars determined by the fundamental method are given. These magnitudes form the basis on which the subsequent comparisons are founded.

TABLE A.

Photographic Magnitudes of 77 Stars within 1° of the Southern Pole.

No.	C.P.D.	No.	*m.* C.P.D.	*m.* Photg.	No.	C.P.D.	No.	*m.* C.P.D.	*m.* Photg.
1	−89°	47	5·7	5·57	41	−88°	114	10·1	11·24
2	89	53	6·5	6·77	42	89	29	10·1	11·24
3	89	34	7·6	7·61	43	89	23	10·1	11·26
4	89	37	8·3	8·33	44	89	40	9·9	11·26
5	88	95	8·5	8·54	45	89	9	9·9	11·28
6	89	22	8·3	8·84	46	89	14	10·0	11·30
7	88	142	8·8	8·88	47	89	57	10·1	11·32
8	89	6	8·9	9·05	48	88	144	10·3	11·38
9	89	3	9·0	9·52	49	89	10	9·9	11·38
10	89	5	9·0	9·62	50	89	31	10·1	11·46
11	89	33	9·0	9·62	51	89	7	10·0	11·46
12	88	46	9·0	9·70	52	89	30	10·1	11·50
13	89	50	9·2	10·02	53	89	44	10·3	11·54
14	89	11	9·2	10·25	54	89	45	10·3	11·54
15	89	12	9·5	10·35	55	89	20	10·4	11·58
16	88	133	9·5	10·41	56	89	49	10·1	11·60
17	89	42	9·9	10·45	57	89	4	10·0	11·60
18	88	132	9·9	10·49	58	89	61	10·1	11·62
19	89	52	9·7	10·51	59	88	121	10·5	11·64
20	89	51	9·7	10·53	60	89	25	10·1	11·66
21	89	16	9·5	10·61	61	89	18	10·2	11·68
22	88	136	9·9	10·63	62	88	37	10·2	11·68
23	89	38	9·5	10·65	63	89	54	10·1	11·70
24	88	138	9·7	10·73	64	89	28	10·3	11·70
25	88	85	9·9	10·73	65	89	60	10·3	11·72
26	88	118	9·9	10·77	66	89	27	10·5	11·74
27	88	40	9·9	10·79	67	89	15	10·2	11·80
28	89	43	9·7	10·79	68	89	58	10·3	11·82
29	89	19	9·9	10·83	69	89	55	10·3	11·82
30	89	35	9·7	10·85	70	88	37	10·1	11·82
31	89	24	9·7	10·85	71	89	13	10·3	11·84
32	89	26	9·9	10·93	72	89	59	10·2	11·88
33	89	56	9·7	10·93	73	89	41	10·3	11·88
34	89	39	9·9	10·99	74	89	36	10·2	11·88
35	88	119	10·0	11·09	75	89	8	10·2	11·92
36	89	21	9·9	11·11	76	89	17	9·9	11·94
37	88	80	10·0	11·11	77	89	46	10·3	11·94
38	88	89	9·9	11·15					
39	89	48	10·0	11·17					
40	88	91	10·0	11·22					

The next step was to compare the magnitudes of stars in selected areas along the parallel -45° declination with those of the polar area. For this purpose the same plate was successively exposed on the selected area and on the South Pole, in each case two exposures of 20^m and 6^m duration being made. In order to equalise as much as possible the effects of atmospheric extinction, the area was photographed when approximately at the altitude of the pole.

Twenty-four areas centred on the R.A. $0^h{\cdot}6$, $1^h{\cdot}6$, etc., were selected. In addition to the comparison with the South Pole, each area was compared in like manner with the two areas six hours preceding and following, so that one and the same area is recorded on three different plates.

From the plates containing the S.P. the relations between the diameters of the polar stars and their photographic magnitudes, taken from Table A, may be derived in the form of graphs (D, m_p) showing the relationship between the diameters D as abscissæ and the magnitudes m_p as ordinates.

Comparing the graphs obtained from different plates, notable differences became apparent, the same diameters corresponding with different magnitudes. On closer inspection, however, it was found that the graphs can be brought into satisfactory coincidence by appropriate shifts in the direction of the m_p-co-ordinate. This result, though not strictly applicable to *single* plates, may at least be accepted for means based on a comparatively small number of plates.

As an instance, Table I. shows the corresponding values of D and m_p found from two groups of four plates, denoted by I and II, for the exposures 20^m and 6^m. Comparing the magnitudes of the four groups I_{20}, I_6, II_{20}, II_6 with their general mean, the differences (I_{20}—mean), etc., are found to be the same for all diameters. Hence, assuming a mean relation (D, m_p) for all the plates, the magnitudes of any plate obtained by entering the mean graph with the observed diameters, while correct in *scale*, require corrections which are constant for the stars of the same plate, but whose amounts vary from plate to plate.

TABLE I.

D.	m_p.				Mean.	Differences from Mean.			
	I_{20}.	I_6.	II_{20}.	II_6.		I_{20}.	I_6.	II_{20}.	II_6.
	m	m	m	m	m	m	m	m	m
6·0	7·36	6·13	6·02	4·99	6·12	+1·24	+0·01	−0·10	−1·13
5·5	7·86	6·63	6·53	5·50	6·63	+1·23	0·00	−0·10	−1·13
5·0	8·38	7·13	7·05	6·02	7·14	+1·24	−0·01	−0·09	−1·12
4·5	8·90	7·65	7·58	6·56	7·67	+1·23	−0·02	−0·09	−1·11
4·0	9·49	8·23	8·17	7·11	8·25	+1·24	−0·02	−0·08	−1·14
3·5	10·10	8·85	8·77	7·70	8·86	+1·24	−0·01	−0·09	−1·16
3·0	10·75	9·49	9·41	8·30	9·49	+1·26	0·00	−0·08	−1·19
2·5	11·40	10·18	10·10	8·98	10·17	+1·23	+0·01	−0·07	−1·19
2·0	12·10	10·92	10·80	9·73	10·89	+1·21	+0·03	−0·09	−1·16

From the twenty-four plates containing the S.P. two mean graphs (D, m_p) were derived for the exposures 20^m and 6^m.

Within each of the twenty-four areas ($-45°$) about forty stars of known C.P.D. magnitudes were selected, and the diameters of their images for both exposures were measured on every plate. With these diameters the mean graphs were entered and the corresponding "tabular" magnitudes were recorded. By a process of successive adjustments the "plate-corrections" were obtained which are to be applied to the tabular magnitudes in order to reduce them to a common zero-point.

The photographic magnitudes thus obtained were then compared with the magnitudes published in the C.P.D. Catalogues. In Table II. the results of this comparison are shown in a somewhat condensed form, three adjacent areas having been combined into means.

TABLE II.

Comparison between C.P.D. and Photographic Magnitudes.

Area: $1^h{\cdot}6$; $-45°$.			$4^h{\cdot}6$; $-45°$.			$7^h{\cdot}6$; $-45°$.			$10^h{\cdot}6$; $-45°$.		
C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.
7·52	8·09	4	7·55	7·52	6	6·30	5·66	2	7·65	7·56	4
8·20	9·07	9	8·27	8·81	7	7·65	7·38	15	8·26	8·27	12
8·71	9·81	14	8·73	9·54	12	8·24	8·41	22	8·69	8·93	22
9·05	10·26	19	9·12	10·33	17	8·73	9·44	17	9·13	9·81	23
9·52	10·84	17	9·60	10·88	26	9·14	10·27	14	9·53	10·62	27
9·87	11·42	36	9·84	11·34	29	9·50	10·71	24	9·94	11·47	15
10·20	11·67	11	10·15	11·62	14	9·87	11·29	6	10·26	12·01	35
						10·22	12·19	31			

Area: $13^h{\cdot}7$; $-45°$.			$16^h{\cdot}7$; $-45°$.			$19^h{\cdot}7$; $-45°$.			$22^h{\cdot}7$; $-45°$.		
C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.	C.P.D.	Ptg.	No. of Stars.
7·52	7·26	6	6·80	6·46	1	6·25	6·04	2	7·70	8·82	8
8·30	8·48	10	7·51	7·10	12	7·54	7·69	13	8·17	9·38	11
8·78	9·46	25	8·22	8·36	14	8·21	8·65	14	8·63	10·05	13
9·08	9·87	22	8·73	9·37	32	8·78	9·47	14	9·16	10·79	22
9·52	10·87	30	9·12	10·19	25	9·15	10·07	14	9·55	11·36	34
9·92	11·55	27	9·54	11·03	29	9·51	10·48	20	9·90	11·60	14
10·27	11·92	17	9·92	11·49	19	9·90	11·14	27			
			10·20	11·77	3	10·28	11·51	18			

The relations between the C.P.D. and photographic magnitudes shown in Table II. may now be utilised for establishing corresponding graphs (D, m_p), D expressing the actinic effects (diameters or densities of the star-images) of the C.P.D. stars as recorded by the observers.

The C.P.D. magnitudes are connected with the quantities D by the formula

$$D = \frac{B}{m} - C,$$

the constants B and C being published for every plate in the Introduction to the C.P.D. Catalogues. Taking the constants referring to the centres of the areas of Table II., the quantities D corresponding with the C.P.D. magnitudes quoted in that table may be calculated. The graphs (D, m_p) for the single areas were found to be similar in character, and can be brought into satisfactory coincidence by appropriate shifts in the direction of the m_p-co-ordinate, a result in agreement with that derived for the selected areas (see Table I.).

By applying the following corrections to the values of m_p in Table II.

h	m	h	m
1·6	+0·33	13·7	−0·17
4·6	+0·27	16·7	+0·31
7·6	−0·34	19·7	+0·35
10·6	−0·34	22·7	−0·12

the individual graphs were reduced to a mean graph (D, m_p) shown in Table III.

Table III.

Relation between D *and the Photographic Magnitude* m_p.

D.	m_p.	D.	m_p.	D.	m_p.	D.	m_p.	D.	m_p.
17	3·64	7·0	6·95	5·0	7·82	3·0	9·06	1·0	10·63
16	3·92	6·8	7·03	4·8	7·92	2·8	9·20	0·8	10·81
15	4·20	6·6	7·11	4·6	8·03	2·6	9·35	0·6	10·99
14	4·50	6·4	7·19	4·4	8·14	2·4	9·50	0·4	11·16
13	4·81	6·2	7·27	4·2	8·25	2·2	9·65	0·2	11·33
12	5·14	6·0	7·35	4·0	8·37	2·0	9·80	0·0	11·50
11	5·48	5·8	7·44	3·8	8·49	1·8	9·95	−0·2	11·68
10	5·83	5·6	7·53	3·6	8·63	1·6	10·12	−0·4	11·85
9	6·20	5·4	7·62	3·4	8·78	1·4	10·26	−0·6	12·03
8	6·58	5·2	7·72	3·2	8·92	1·2	10·45	...	...

The quantities D represent the *diameters* of the star-images down to about 1·7. For fainter objects, however, D denotes the *densities* of the images expressed on an empirical scale. The transition from the one actinic effect to the other is marked by a distinct change in the character of the curve.

For D > 1·7 the relation (D, m_p) is well satisfied by the equation

$$m_p = 12{\cdot}98 - 2{\cdot}266\sqrt{D}.$$

For this reason the values of m_p corresponding with diameters

> 11, which are outside the range of the observations recorded in Table II., have been computed from this formula.

For $D < 1{\cdot}7$, on the other hand, the linear relation

$$m_p = 11{\cdot}50 - 0{\cdot}865\,D$$

is in excellent agreement with the observations.

The quantities B and C show considerable variations from plate to plate. It is found, however, that approximately

$$C = \frac{B}{10} + 0{\cdot}1$$

may be assumed. Hence, if the tabular C.P.D. magnitude is denoted by m, we may write:

$$D = B\frac{10 - m}{10m} - 0{\cdot}1.$$

Computing D for given values of m and B, and taking the corresponding photographic magnitudes m_p from Table III., the relation between m (C.P.D.) and m_p can be brought into the form of Table IV.

TABLE IV.

m (C.P.D.)	Photographic Magnitudes m_p.											
	B = 100.	110.	120.	130.	140.	150.	160.	170.	180.	190.	200.	210.
5·5	6·55	6·24	5·93	5·67	5·38	5·10	4·82	4·57	4·32	4·08	3·85	3·63
6·0	7·14	6·86	6·62	6·37	6·12	5·88	5·66	5·42	5·19	4·97	4·74	4·54
6·5	7·68	7·43	7·21	6·99	6·78	6·59	6·39	6·18	5·98	5·80	5·61	5·43
7·0	8·26	8·02	7·80	7·59	7·40	7·22	7·05	6·89	6·73	6·56	6·40	6·24
7·5	8·90	8·65	8·43	8·23	8·05	7·87	7·70	7·55	7·40	7·24	7·13	6·99
8·0	9·50	9·31	9·13	8·95	8·78	8·60	8·43	8·28	8·14	8·00	7·87	7·74
8·5	10·06	9·92	9·79	9·66	9·52	9·39	9·26	9·13	9·00	8·88	8·76	8·62
9·0	10·62	10·52	10·42	10·33	10·23	10·14	10·05	9·96	9·88	9·78	9·71	9·63
9·5	11·14	11·09	11·05	11·01	10·95	10·90	10·86	10·82	10·77	10·72	10·67	10·62
10·0	11·59	11·59	11·59	11·59	11·59	11·59	11·59	11·59	11·59	11·59	11·59	11·59

It is important to examine how far the observed values of Table II. can be represented by Table IV.

The values of B for the eight mean areas of Table II. are—

Area. h	B.	Area. h	B.
1·6	128·6	13·7	184·7
4·6	170·0	16·7	187·1
7·6	194·1	19·7	140·9
10·6	198·0	22·7	141·0

Taking m_p from Table IV. for the values of m (C.P.D.) shown in Table II., and comparing these tabular values with the observed m_p of Table II., it is found that in all cases the *scale* is correctly represented, while the following corrections for zero-point are derived :—

TABLE V.

Area. h	Δm.	Area. h	Δm.
1·6	−0·15	13·7	0·00
4·6	−0·02	16·7	+0·09
7·6	+0·09	19·7	−0·39
10·6	−0·10	22·7	+0·35

To show the accuracy with which the photographic magnitudes derived from Table IV. represent the observations, it will suffice to quote the results of the two areas $1^h{\cdot}6$ and $10^h{\cdot}6$, for which B has the extreme values 128·6 and 198·0 respectively.

Area: $1^h{\cdot}6$; B=128·6.					Area: $10^h{\cdot}6$; B=198·0.				
m. (C.P.D.).	m_p. II.	m_p. IV.	Diff. (II.–IV.).	No. of Stars.	m. (C.P.D.).	m_p. II.	m_p. IV.	Diff. (II.–IV.).	No. of Stars.
7·52	8·09	8·30	−0·21	4	7·65	7·56	7·37	+0·19	4
8·20	9·07	9·25	−0·18	9	8·26	8·27	8·34	−0·07	12
8·71	9·81	9·96	−0·15	14	8·69	8·93	9·15	−0·22	22
9·05	10·26	10·41	−0·15	19	9·13	9·81	9·96	−0·15	23
9·52	10·84	11·04	−0·20	17	9·53	10·62	10·73	−0·11	27
9·87	11·42	11·45	−0·03	36	9·94	11·47	11·49	−0·02	15
10·20	11·67	11·81	−0·14	11	10·26	12·01	12·06	−0·05	35

The corrections Δm of Table V. show no pronounced dependence on the position of the area. For statistical purposes, where mean results derived from a number of plates are considered, the magnitudes taken from Table IV. may therefore be considered systematically correct both in scale and zero-point. This result has been also verified by comparison with a different series of observations. In vol. 71, No. 2, of the *Annals of Harvard Observatory*, the late Mrs. Fleming published the photographic magnitudes of stars in standard regions, nine of which are situated along the parallel 45° south declination. These magnitudes were derived by correcting the visual Harvard magnitudes for colour, according to the spectral type. It will be pointed out later on that Mrs. Fleming's system differs somewhat from the Cape system here adopted, but the differences do not affect the present question.

The standard regions are marked by the letters E_1 . . . E_9. Taking the constants B and C from the introductory table of *Cape Annals*, vol. iv., the diameters D of the stars were calculated by

means of the C.P.D. magnitudes, and the corresponding photographic magnitudes (Cape system) were taken from Table III. The mean differences (Harvard – Cape) were found as follows :—

TABLE VI.

Area.	R.A. h	(Harv. – Cape). Δm.
E_1	1·3	0·00
E_2	4·0	−0·09
E_3	6·7	−0·04
E_4	9·3	+0·37
E_5	12·0	−0·08
E_6	14·7	−0·02
E_7	17·3	−0·10
E_8	19·0	+0·17
E_9	22·7	+0·15

The differences are of the same order as those of Table V., and show no distinct dependence on the position of the area.

The probable error of a single magnitude is $\pm 0^m{\cdot}27$; this includes the error in the C.P.D. measures, in the photometric measures, and in the type corrections; further, the error arising from the particular condition of the plates and from irregularities in atmospheric extinction. The two last-named sources of errors are responsible for the plate corrections shown in Tables V. and VI. From these the p.e. of a *single* plate would appear to be about $\pm 0^m{\cdot}19$. This means that by deriving the photographic magnitudes of a single plate of the C.P.D. by means of Table II. and the constants B, C, the zero-points of these magnitudes are determined with a p.e. $\pm 0{\cdot}19$.

A similar result was obtained from the discussion of the selected areas in $-45°$ decl. If the magnitudes of the individual plates are determined from a mean graph (D, m_p), the p.e. of the zero-point of these magnitudes is found to be $\pm 0^m{\cdot}16$. This "plate-error," however, is reduced to $\pm 0^m{\cdot}07$ if the magnitudes are derived from those of the polar area photographed on the same plate.

The photographic magnitudes m_p, as already mentioned, are based on a fundamental system derived for a sequence of stars in the vicinity of the South Pole. It is important to examine the relation of this system to other photometric systems. For this purpose the following comparisons were made—

(*a*) with the Harvard *visual* system;
(*b*) with the *photographic* Harvard system derived from stars of known spectral types.

Professor Kapteyn kindly placed at our disposal a list of comparisons of the C.P.D. magnitudes with the Harvard visual

magnitudes for stars within the zone 38°–53° S. decl. From his tables the differences (Harvard visual – C.P.D.) arranged in order of C.P.D. magnitudes were determined as shown in Table VIIA.

On the other hand, the lists E_1 to E_9, published in *Harvard Annals*, 71, No. 2, show the "photographic" magnitudes of stars with known spectra in selected areas situated along the parallel 45° S. decl. As mentioned before, they are based on *visual* magnitudes to which the corrections for type were added. The differences (Harvard phot. – C.P.D.) are shown in Table VIIB. in order of C.P.D. magnitudes.

TABLE VII.

A.	*m* (C.P.D.).	(Harv. vis. – C.P.D.).	B.	*m* (C.P.D.).	(Harv. phot. – C.P.D.).	No. of Stars.
	6·5	−0·93		4·95	−0·91	12
	7·0	−0·78		6·59	−0·38	26
	7·5	−0·49		7·21	+0·03	58
	8·0	−0·21		7·66	−0·02	34
	8·5	+0·06		8·21	+0·54	44
	9·0	+0·31		8·74	+0·79	46
	9·5	+0·52		9·28	+0·88	33
	10·0	+0·71				

If in Tables A and B the figures corresponding with the same values of *m* (C.P.D.) are compared, we obtain the following differences—Harvard (phot. – vis.):

TABLE VIII.

Harvard (phot. – vis.).

m (C.P.D.).	Harvard (phot. – vis.).
6·59	+0·52
7·21	+0·68
7·66	+0·42
8·21	+0·64
8·74	+0·61
9·28	+0·45

The figures of the second column express the average colour-indices due to spectral type. Within the range of Table VIII., at any rate, this colour-index appears to be independent of the apparent magnitude. Tables VIIA. and B are therefore brought into agreement by adding 0·55 to the second column in the former. From the combined graph (A and B) the differences (Harv. phot. – C.P.D.) of the second column of Table IX. were derived.

On the other hand, from the figures of Table II., combined for all the stars of the zone, the differences (Cape phot. – C.P.D.) shown in the third column of Table IX. were obtained.

TABLE IX.

m (C.P.D.).	(Harv. phot. − C.P.D.)	(Cape phot. − C.P.D.)	(Cape − Harvard.)
6·5	−0·40	−0·39	+0·01
7·0	−0·19	−0·19	0·00
7·5	+0·04	+0·04	0·00
8·0	+0·31	+0·32	+0·01
8·5	+0·59	+0·66	+0·07
9·0	+0·83	+1·00	+0·17
9·5	+1·04	+1·29	+0·25
10·0	+1·24	+1·54	+0·30

The figures of the last column show the differences between the system of photographic magnitudes of the Cape south polar sequence and the Harvard photographic system based on visual magnitudes corrected for "type." The two systems agree well for the brighter stars, but a gradually increasing discrepancy is noticeable in the fainter stars. This phenomenon points either to the presence of systematic errors depending on magnitude in the systems compared, or to changes of the colour-index due to an optical cause other than that indicated by the type of spectrum only (selective space absorption, for instance).

Since the Harvard photographic system is only *indirectly* derived through the medium of the visual magnitudes, it is not strictly comparable with the Cape system, which is based on direct photographic observations. No other photographic system being at present available for stars of the southern hemisphere, a comparison had to be made with the system of northern photographic magnitudes based on the north polar sequence published in *H.C.*, 170. An attempt was made to effect this comparison by means of the relation between magnitudes and star numbers.

For the northern stars the relation between m_p (limiting photographic magnitude) and B_m (number of stars contained within 1 square degree and brighter than m_p) has been statistically examined by Messrs. Chapman and Melotte (*Memoirs of the R.A.S.*, **60**, part iv.). For the southern stars the counts of Professor Stratonoff (*Publications de l'Observatoire astronomique et physique de Tachkent*, No. 3) are at our disposal. The limiting magnitudes in Stratonoff's investigations are the C.P.D. magnitudes 6·0, 6·5 . . . 9·5. By means of the constants B and C published in the Introductions to the C.P.D. Catalogues on the one hand, and Table III. on the other, these magnitudes can be reduced to the Cape photographic system. In this manner the relation between B_m and m_p (Cape photographic) can be found for any desired region.

It must be considered, however, that Table III., showing the relation between the sizes (or densities) of the star-images and the magnitudes, naturally depends on the optical conditions of the instrument employed. These conditions unfortunately are not uniform throughout the whole extent of the C.P.D. The stars south of the parallel − 58° were observed with the 6-inch Dallmeyer lens in its original form, while the zones north of − 58° were

photographed at a later epoch with the same lens after it had been repolished by the maker. Table III., being derived from stars in $-45°$ decl., therefore applies to the latter observations only.

The material was subdivided into three zones centred on $-25°$, $-35°$, and $-45°$ declination, and within each zone a further subdivision into two groups was made according to galactic latitude. Table X. shows the results for each zone and group separately. The first column gives the limiting magnitude (C.P.D.), the third column shows the corresponding photographic magnitude m_p (Cape system), and the second column exhibits the logarithms of B_m found from Stratonoff's statistical material.

For each latitude group the relation ($\log B_m$, m_p) was represented in the form of a graph with co-ordinates $\log B_m$ and m_p separately for every zone. It was found that these graphs could be brought into coincidence by slightly changing the zero-points of the m_p-co-ordinates. Applying the necessary corrections for each latitude group, mean graphs were derived which are shown in tabular form in Table XI. The zero-points of m_p were so chosen that in the final comparison the graphs agree as closely as possible with the corresponding graphs of Chapman and Melotte.

The fourth columns of Table X. show the values of m_p obtained by entering the graphs of Table XI. with the argument $\log B_m$ of the second columns, and the differences Δm_p (observed – tabular) are exhibited in the fifth columns.

TABLE X.

I. *Galactic Latitude* 30°–90°; *Mean Latitude* 55°.

Zone $-25°$. B=150·9, C=15·3.					Zone $-35°$. B=135·1, C=13·4.				
m (C.P.D.).	log B_m.	m_p.	Table XI.	Δm_p.	m (C.P.D.).	log B_m.	m_p.	Table XI.	Δm_p.
6·5	−1·14	6·62	6·48	+0·14	6·5	−1·09	6·80	6·58	+0·22
7·0	−0·79	7·23	7·14	+0·09	7·0	−0·71	7·40	7·28	+0·12
7·5	−0·45	7·92	7·78	+0·14	7·5	−0·41	8·03	7·86	+0·17
8·0	−0·04	8·63	8·60	+0·03	8·0	−0·06	8·71	8·59	+0·22
8·5	+0·28	9·42	9·37	+0·05	8·5	+0·22	9·42	9·22	+0·20
9·0	+0·62	10·20	10·27	−0·07	9·0	+0·54	10·12	10·05	+0·07
9·5	+0·85	10·99	10·93	+0·06	9·5	+0·76	10·81	10·68	+0·13
			Mean	+0·06				Mean	+0·16

Zone $-45°$. B=143·5, C=14·4.				
m (C.P.D.).	log B_m.	m_p.	Table XI.	Δm_p.
6·5	−1·09	6·69	6·58	+0·11
7·0	−0·70	7·31	7·30	+0·01
7·5	−0·35	7·98	7·98	0·00
8·0	−0·03	8·71	8·62	+0·09
8·5	+0·24	9·42	9·27	+0·15
9·0	+0·56	10·20	10·10	+0·10
9·5	+0·78	10·90	10·74	+0·16
			Mean	+0·09

II. *Galactic Latitude* 0°–30°; *Mean Latitude* 15°.

Zone −25°. B=175·2, C=17·6.					Zone −35°. B=170·0, C=17·2.				
m (C.P.D.).	log Bm.	m_p.	Table XI.	Δm_p.	m (C.P.D.).	log Bm.	m_p.	Table XI.	Δm_p
6·5	−0·93	6·06	6·20	−0·14	6·5	−1·02	6·20	6·02	+0·18
7·0	−0·66	6·80	6·74	+0·06	7·0	−0·67	6·76	6·72	+0·04
7·5	−0·32	7·44	7·44	0·00	7·5	−0·29	7·58	7·50	+0·08
8·0	+0·08	8·20	8·25	−0·05	8·0	+0·11	8·30	8·31	−0·01
8·5	+0·45	9·06	9·06	0·00	8·5	+0·44	9·20	9·03	+0·17
9·0	+0·86	9·89	10·01	−0·12	9·0	+0·82	10·04	9·91	+0·13
9·5	+1·19	10·81	10·88	−0·07	9·5	+1·14	10·90	10·75	+0·15
			Mean	−0·05				Mean	+0·12

Zone −45°. B=188·9, C=18·8.				
m (C.P.D.).	log Bm.	m_p.	Table XI.	Δm_p.
6·5	−0·94	5·76	6·18	−0·42
7·0	−0·61	6·50	6·84	−0·34
7·5	−0·25	7·19	7·58	−0·39
8·0	+0·17	7·92	8·43	−0·51
8·5	+0·51	8·78	9·19	−0·41
9·0	+0·90	9·65	10·11	−0·46
9·5	+1·21	10·54	10·94	−0·40
			Mean	−0·42

TABLE XI.

log Bm.	m_p.		log Bm.	m_p.	
	55°.	15°.		55°.	15°.
−1·20	6·36	...	0·00	8·68	8·08
−1·10	6·56	...	+0·10	8·92	8·29
−1·00	6·75	6·07	+0·20	9·17	8·50
−0·90	6·94	6·26	+0·30	9·42	8·71
−0·80	7·12	6·45	+0·40	9·67	8·94
−0·70	7·30	6·66	+0·50	9·94	9·17
−0·60	7·49	6·87	+0·60	10·21	9·39
−0·50	7·68	7·08	+0·70	10·50	9·62
−0·40	7·88	7·28	+0·80	10·79	9·86
−0·30	8·08	7·48	+0·90	...	10·11
−0·20	8·27	7·68	+1·00	...	10·37
−0·10	8·47	7·88	+1·10	...	10·64
			+1·20	...	10·91

Table XI. enables us to compare the magnitudes of the Cape system with those of the north polar sequence on the supposition that for regions of the same galactic latitude the same star numbers

B_m correspond with the same limiting magnitudes in both hemispheres.

From Table VII. of Chapman and Melotte's paper the correlated values of m_p and log B_m were taken for Zone VI. (55° gal. lat.) and Zone II. (15° gal. lat.). They are shown in the first and third columns of Table XII., while the second columns give the limiting magnitudes referred to the Mount Wilson photographic system. On the other hand, from Table XI. for the values of log B_m the corresponding magnitudes m_p (Cape) shown in the fourth columns were extracted. The fifth columns give the differences Δm_p (Mount Wilson – Cape).

TABLE XII.

Galactic Latitude 55°.					Galactic Latitude 15°.				
m_p. (H.C. 170).	m_p. (Mt. Wilson).	log B_m. (C. & M.).	m_p. (Cape).	Δm_p. (Mt. W. – Cape).	m_p. (H.C. 170).	m_p. (Mt. Wilson).	log B_m. (C. & M.).	m_p. (Cape).	Δm_p. (Mt. W. – Cape)
6·00	5·98	−1·39	5·98	0·00	6·00	5·98	−1·05	5·97	+0·01
7·00	7·02	−0·83	7·07	−0·05	7·00	7·02	−0·50	7·08	−0·06
8·00	8·09	−0·31	8·06	+0·03	8·00	8·09	+0·01	8·12	−0·03
9·00	9·18	+0·17	9·10	+0·08	9·00	9·18	+0·49	9·15	+0·03
10·00	10·25	+0·61	10·24	+0·01	10·00	10·25	+0·94	10·21	+0·04
10·50	10·77	+0·81	10·82	−0·05	10·50	10·77	+1·14	10·74	+0·03
			Mean	0·00				Mean	0·00

The residuals Δm_p show that the *scale* of the Cape system is in satisfactory agreement with that of the Mount Wilson system for both high and low galactic latitudes. The differences in zero-point are indicated by the mean values of Δm_p of Table X. Here the only serious discrepancy is shown by the low latitude group of zone −45°. The fact demonstrated in Table V., that the zero-points of the magnitudes determined by Table III. show no systematic errors depending on the position when compared with the magnitudes determined from direct comparisons with the polar sequence, makes it unlikely that this discrepancy is due to an erroneous determination of the magnitudes. It probably points to a cosmical cause through which, in the particular region of the galaxy intersected by the parallel −45° decl., the stars appear photographically brighter.

The results shown in Tables X. and XII. justify the conclusion that the system of photographic magnitudes of the south polar sequence given in Table A agrees in scale and zero-point with the system of the north polar sequence based on the Mount Wilson observations.

Royal Observatory, Cape of Good Hope:
1917 *August.*

On the Relation between Star Counts and Apparent Magnitudes.
By J. Halm.

§ 1. Mathematical Theory.

The relation between the apparent magnitude, m, and the number of stars, B_m, brighter than m, counted in a field of unit area (1 square degree), can be derived from the following general considerations:—

Let ν denote the total number of stars contained in unit volume at any point of space, P, whose distance from the observer is ρ.

P is completely defined by ρ and by spherical co-ordinates (λ, β), for which the galactic longitude and latitude may be chosen.

The *frequency*, ν, is considered an unknown function of the co-ordinates (ρ, λ, β).

Further, provided that the medium through which the star's light is propagated has *no* influence on the intensity of radiation, the magnitude m of a star at distance ρ would be changed to M, if the star were brought to unit distance (1 parsec), where

$$M = m - 5 \log \rho \quad . \quad . \quad . \quad . \quad (1)$$

M being defined as the absolute magnitude.

More generally the possibility of an "absorbing" effect of the transmitting medium must be admitted, and M will therefore be defined by the relation

$$M = m - 5 \log \rho - \psi \quad . \quad . \quad . \quad . \quad (1a)$$

where ψ is considered an unknown function of the co-ordinates (ρ, λ, β), depending solely on the *optical* properties of the *medium*.

The ν stars contained in unit volume at P possess different luminosities, *i.e.* different values of M. Let the number of stars whose absolute magnitudes lie between $(M \pm \frac{1}{2}dM)$ be generally expressed by

$$\nu\phi(M)dM \quad . \quad . \quad . \quad . \quad (2)$$

the function $\phi(M)$ being defined as the luminosity law.

In accordance with the views of Kapteyn, Schwarzschild, and others, the assumption is introduced that the luminosity law can be expressed in the form

$$\phi(M) = \frac{h}{\sqrt{\pi}} e^{-h^2[M-\bar{M}]^2} \quad . \quad . \quad . \quad . \quad (3)$$

which implies that the absolute magnitudes of the individual stars surrounding P are distributed fortuitously about an average value $\bar{M}$.

The reciprocal of $(h\sqrt{2})$ will then define the "mean error" or "spread" of the individual M's as compared with the average $\bar{M}$. The subsequent deductions are based on the assumption that h is independent of the position of the point P.

On the other hand, no similar conditions are assumed with regard to $\bar{M}$. The restriction that $\bar{M}$ also should be independent of the position, which is an essential feature of the theories advanced by Kapteyn, Schwarzschild, and Seeliger, is now abandoned, and $\bar{M}$ will be regarded as an unknown function of the co-ordinates (ρ, λ, β).

Accordingly, out of the total number of stars, ν, contained within a unit volume centred at P,

$$\frac{h}{\sqrt{\pi}}\nu e^{-h^2[M-\bar{M}]^2}dM \quad . \quad . \quad . \quad . \quad (4)$$

stars have absolute magnitudes between $(M \pm \frac{1}{2}dM)$, M being defined by equation $(1a)$, and ν and $\bar{M}$ being unknown functions of (ρ, λ, β).

Now, the volume of a spherical shell contained between the radii $(\rho \pm \frac{1}{2}d\rho)$ is

$$4\pi\rho^2 d\rho,$$

and the volume of the part of the shell contained within any solid angle, $d\omega$, whose apex is at the point of observation, is

$$\rho^2 d\rho d\omega,$$

the solid angle being defined by the area of the field, expressed in angular measure, within which the stars are counted.

Consequently, the number of stars found within a field $d\omega$, whose distances from the observer lie between $(\rho \pm \frac{1}{2}d\rho)$, and whose absolute magnitudes are between $(M \pm \frac{1}{2}dM)$, is expressed by

$$dB = \frac{h}{\sqrt{\pi}}\nu e^{-h^2[M-\bar{M}]^2}\rho^2 d\rho d\bar{M} d\omega \quad . \quad . \quad . \quad (5)$$

Regarding ρ, λ, β as constant within any elementary volume $d\rho d\omega$, the formula $(1a)$ gives

$$dM = dm,$$

and hence the number of stars contained within a field $d\omega$, whose apparent magnitudes lie between $(m \pm \frac{1}{2}dm)$, and whose distances are between $(\rho \pm \frac{1}{2}d\rho)$, is

$$dB = \frac{h}{\sqrt{\pi}}\nu e^{-h^2[m-5\log\rho-\psi-\bar{M}]^2}\rho^2 d\rho dm d\omega \quad . \quad . \quad (6)$$

In place of the function ν let us now introduce a new variable, r, which satisfies the condition

$$\nu = \nu_0 \frac{r^2 dr}{\rho^2 d\rho} \quad . \quad . \quad . \quad . \quad (7)$$

where ν_0 is a constant, and a further function, f, where

$$\left(\frac{\rho}{r}\right)^5 = 10^f \quad . \quad . \quad . \quad . \quad (8)$$

The quantities r, f thus introduced depend solely on the space-distribution of stars, and in no way on their luminosities.

Substituting into (6) we obtain

$$dB = \frac{h}{\sqrt{\pi}} \nu_0 e^{-h^2[m-5\log r-(f+\phi+\psi)]^2} r^2 dr dm d\omega \quad . \quad . \quad (9)$$

if we further write

$$\bar{M} = \phi \quad . \quad . \quad . \quad . \quad . \quad (10)$$

where ϕ signifies an unknown function of (r, λ, β).

If f, ϕ, ψ are known, the double integration between the limits $(\rho = 0, \infty)$ and $m = (-\infty, m)$ gives the number of stars, B_m, brighter than the limiting magnitude m in a field $d\omega$.

Equation (9) represents the fundamental equation of stellar statistics in the most general form which can be assigned to it on the assumption that the luminosity law has the analytical form (3).

To effect its integration definite knowledge of the analytical form of the functions f, ϕ, ψ is required. In this respect a result obtained in a previous paper (*M.N.*, **77**, 243–280) is of importance. It was shown that the relation between B_m and m, found statistically by Messrs. Chapman and Melotte from a discussion of results obtained from the Franklin-Adams plates, was satisfactorily represented by integrating between $r = (0, \infty)$ and $m = (-\infty, m)$ the differential expression

$$dB = \frac{h}{\sqrt{\pi}} \nu_0 e^{-h^2[m-5\log r-\alpha r-\bar{M}_0]^2} r^2 dr dm d\omega \quad . \quad . \quad (11)$$

where $\bar{M}_0$ is independent of r.

Comparing (11) and (9) we have

$$f + \phi + \psi = \alpha r + \bar{M}_0 \quad . \quad . \quad . \quad . \quad (12)$$

Hence the functions f, ϕ, ψ may be assumed linear in r, and may be stated in the general form

$$\left.\begin{aligned} f &= \alpha_f r + a_f \\ \phi &= \alpha_\phi r + a_\phi \\ \psi &= \alpha_\psi r + a_\psi \\ \alpha &= \alpha_f + \alpha_\phi + \alpha_\psi \\ \bar{M}_0 &= a_f + a_\phi + a_\psi \end{aligned}\right\} \quad . \quad . \quad . \quad (13)$$

where the α's and a's are functions of the spherical co-ordinates (λ, β).

The function f defines the *frequency* law, ϕ determines the *luminosity* law, and ψ indicates the character of the influence exerted by the propagating medium on the radiative energies (space-absorption).

Without further criteria to distinguish between the component parts of the α's and a's, definite conclusions regarding the space-distribution of stars and the law of luminosities are impossible.

Whenever such conclusions have been drawn in the past, it was done on the assumption that the α coefficients in two of the three functions are zero.

Thus Schwarzschild's and Kapteyn's investigations are based on the assumptions $\phi = \text{const.}$, $\psi = 0$, and consequently $\alpha = \alpha_f$, so that in this case a definite frequency law can be derived.

On the other hand, in my previous paper the view was advocated that $f = 0$, $\phi = \text{const.}$, $\alpha = \alpha_\psi$; *i.e.* that the frequencies and average luminosities are the same in all parts of space, but that there is an appreciable amount of space-absorption.

Both views are perfectly reconcilable with the form of the fundamental equation (11), and hence must lead to the same conclusions regarding the relation between B_m and m. But different results are obtained with regard to the relation between the true distances ρ and the parameters r.

In the former case, according to equation (8),

$$\log \rho = \log r + \tfrac{1}{5}(\alpha r + \alpha_f),$$

while in the latter case we have

$$\log \rho = \log r.$$

This difference in the interpretation of ρ in terms of r suggests at once a criterion which will enable us to separate the coefficient α_f from the others.

Let us suppose that the constants ν_0, h, α, $\bar{M}_0$ of equation (11) have been determined so that the observed relation (B_m, m) is satisfied. Let further the average reciprocals $\left(\frac{1}{r}\right)$ of stars of the same apparent magnitude be computed by performing the integrations involved in the expression

$$\left(\frac{1}{r}\right) = \frac{\int r^{-1} d\mathrm{B}}{\int d\mathrm{B}},$$

the integration being effected for constant m and between the limits $r = (0, \infty)$.

According to equations (8) and (13), the relation between ρ and r is generally

$$\left.\begin{aligned} \log \rho &= \log r + \tfrac{1}{5}(\alpha_f r + \alpha_f) \\ \text{or} \qquad \log \left(\frac{1}{\rho}\right) &= \log \left(\frac{1}{r}\right) - \tfrac{1}{5}(\alpha_f r + \alpha_f) \end{aligned}\right\} \quad . \quad . \quad (14)$$

Now from the parallactic motions the average parallaxes $\left(\frac{1}{\rho}\right)$ of stars of apparent magnitude m can be determined. Hence it is possible to determine α_f and α_f so that the average parallaxes fulfil the condition (14).

Further, Sir Frank Dyson and Professor Eddington have shown how, from the transverse *motus peculiares*, the distribution in distance of stars within the same limits of apparent magnitude

may be obtained. From their formula may be found the proportion of the total number of such stars as are enclosed within any sphere of radius ρ. On the other hand, from equation (11) for the same magnitudes the proportion enclosed within a sphere of radius r may be derived.

Comparing ρ and r for equal proportions, equation (14) will again determine α_f and α_f.

Since the frequency law depends exclusively on these two coefficients, the space-distribution of stars may thus be found.

Researches of this character naturally require a great refinement in the observational data which we can scarcely claim to have attained at present. Nevertheless, though accurate determinations of α_f and α_f may not yet be expected from the evidence of the proper motions, at least some idea may be formed as to whether the observations demand the coefficients α_ϕ and α_ψ, or whether they can be sufficiently represented by the condition $\alpha_f = a$, $\alpha_\phi = \alpha_\psi = 0$, on which Kapteyn's and Schwarzschild's theories are based.

The presence of coefficients of the type α_ϕ and α_ψ may also be traced by means of another criterion of an entirely different complexion. Star magnitudes are determined from observations of a limited portion of the radiative energy only, viz. that which reacts on the receiving apparatus—on the retina of the eye in visual observations, and on the film of the plate in photographic observations. In these two cases we therefore operate with light of different effective wave-lengths. Now, by observation, the relations between B_m on the one hand, and m_v (visual) and m_p (photographic) on the other, may be established, and accordingly the constants of equation (11) may be determined which satisfy these particular relations. Thus we obtain values of the moduli a_v and a_p.

But since ν, which depends on the α_f terms only, cannot be a function of the effective wave-length, it follows that under the condition ($\alpha_f = a$, $\alpha_\phi = \alpha_\psi = 0$) the moduli a_v and a_p must be identical. Otherwise different frequency laws would be obtained according to whether visual or photographic observations are selected. On the other hand, if α_ϕ and α_ψ are sensible quantities due to optical causes depending on the wave-length—such as, for instance, selective space-absorption or change of type with distance,—then, on account of the fourth equation (13), a_v and a_p could have different values.

It is the object of the subsequent part of this paper to examine the evidence of the observations with regard to the presence of α_ϕ and α_ψ terms. The preceding remarks suggest the following method of procedure:—

First. It is required to show that equation (11), by integration between the limits $r = (0, \infty)$ and $m = (-\infty, m)$, leads to relations between B_m and m which, with suitably chosen quantities ν_0, h, a, and $\bar{M}_0$, represent the observed relations in any part of the sky.

Secondly. Having derived separate values of a_v and a_p, the differences between these moduli have to be examined with the view of ascertaining whether they can be explained by α_f terms

only, or whether the existence of terms a_ϕ and a_ψ, depending on the effective wave-lengths, must be accepted.

Thirdly. For the same purpose the true distances ρ obtained from proper motions are to be compared with the parameters r derived from equation (12).

Before entering upon a discussion of the observations, however, a simple method may be pointed out by which equation (12) can be integrated for any values of α and $\bar{M}_0$, if ν_0 and h are universal constants.

If in (11) we substitute

$$r = \frac{\alpha'}{\alpha} \cdot r' \quad . \quad . \quad . \quad . \quad . \quad (15)$$

we obtain

$$dB' = \frac{h}{\sqrt{\pi}} \nu_0 r'^2 dr' e^{-h^2[m'-5\log r'-\alpha' r'-\bar{M}_0']^2} dm' d\omega \quad . \quad (16)$$

when the following relations are satisfied:

$$\left.\begin{aligned} \log r' &= \log r - \log(\alpha'/\alpha) \\ \log B_{m'} &= \log B_m - 3\log(\alpha'/\alpha) \\ m' &= m - 5\log(\alpha'/\alpha) - (\bar{M}_0 - \bar{M}_0') \end{aligned}\right\} \quad . \quad . \quad (17)$$

Representing any relation (B_m, m) in the form of a graph whose co-ordinates are $\log B_m$ and m, the graphs obtained for different values of α and $\bar{M}_0$ are therefore similar in form and differ only in the zero-points of their co-ordinates. Any graph may thus be brought into coincidence with one particular graph derived for special values α' and $\bar{M}_0'$ by appropriate shifts parallel to the co-ordinate axes. Determining in this manner the differences $(m' - m)$ and $(\log B_{m'} - \log B_m)$, the quantities α and $\bar{M}_0$ are found from equations (17).

Now in table ix. of my previous paper a special case of integration of (16) with the constants

$$\nu_0 = 0{\cdot}07343\,; \quad h = 0{\cdot}249\,; \quad \alpha' = 0{\cdot}00305\,; \quad \bar{M}_0' = 3{\cdot}95$$

has already been given. This special graph may therefore be used for finding the values of α and $\bar{M}_0$ in any region of the sky, provided that the same values of ν_0 and h are applicable to all the regions.

The following Table I. is mainly a reproduction of the figures of table ix., *loc. cit.* It shows the correlated values of m', $\log B_{m'}$, $\log \bar{r}_{m'}$, $\log \bar{p}_{m'}$, and $\log \bar{\pi}_{m'}$.

The values of $\log B_{m'}$ were obtained by numerical integration of (16) between the limits $r' = (0, \infty)$, $m' = (-\infty, m')$.

The quantities $\bar{r}_{m'}$ represent the *average* parameters r' of stars of magnitude m', and the quantities $\bar{p}_{m'}$ are the *average* reciprocals $(1/r')$ of stars of magnitude m'. They were obtained by numerical integrations of the expressions

$$\bar{r}_{m'} = \frac{\int r' dB'}{\int dB'}\,; \qquad \bar{p}_{m'} = \frac{\int 1/r' dB'}{\int dB'}\,,$$

the integrations being effected for constant m' between the limits $r' = (0, \infty)$.

The quantities $\bar{r}_m$ and $\bar{p}_m$ are proportional to the true *average* distances and parallaxes in the special case $\alpha_f = 0$; $\alpha_\phi + \alpha_\psi = \alpha$.

Lastly, the quantities $\bar{\pi}_m'$ represent the *average* reciprocals of $[r' \,.\, 10^{\frac{1}{5}\alpha' r'}]$; they were calculated from the expressions

$$\bar{\pi}_m' = \frac{\int \frac{10^{-\frac{1}{5}\alpha' r'}}{r'} dB'}{\int dB'},$$

the integration being effected for constant m between the limits $r' = (0, \infty)$.

The parameters $[r \,.\, 10^{\frac{1}{5}\alpha r}]$ are proportional to the true distances in the special case $\alpha = \alpha_f$; $\alpha_\phi + \alpha_\psi = 0$.

TABLE I.

m'.	$\log B_{m'}$.	$\log \bar{r}_{m'}$.	$\log \bar{p}_{m'}$.	$\log \bar{\pi}_{m'}$.	m'.	$\log B_{m'}$.	$\log \bar{r}_{m'}$.	$\log \bar{p}_{m'}$.	$\log \bar{\pi}_{m'}$.
1·5	−3·456	1·737	−1·237	−1·250	11·5	1·308	2·630	−2·443	−2·595
2·5	−2·908	1·856	−1·395	−1·412	12·5	1·673	2·696	−2·526	−2·710
3·5	−2·368	1·965	−1·546	−1·570	13·5	2·017	2·757	−2·607	−2·827
4·5	−1·841	2·067	−1·686	−1·715	14·5	2·340	2·819	−2·682	−2·942
5·5	−1·333	2·163	−1·817	−1·856	15·5	2·643	2·878	−2·753	−3·058
6·5	−0·842	2·253	−1·939	−1·989	16·5	2·925	2·934	−2·822	−3·180
7·5	−0·371	2·337	−2·053	−2·118	17·5	3·188	2·988	−2·888	−3·304
8·5	+0·081	2·417	−2·160	−2·241	18·5	3·436	3·040	−2·951	−3·431
9·5	+0·511	2·491	−2·260	−2·361	19·5	3·663	3·091	−3·011	−3·561
10·5	+0·920	2·562	−2·354	−2·478	20·5	3·875	3·141	−3·068	−3·696

The following approximate formulæ, which are sufficiently accurate within a considerable range of m' (about 4^m to 17^m), will be found useful in subsequent calculations:

$$m' - \bar{M}_0' = 2{\cdot}98 + \tfrac{5}{3} \log B_m' + 1{\cdot}796\, \alpha' \bar{r}_m' \quad . \quad . \quad (18)$$

$$\log B_m' = -4{\cdot}397 + 0{\cdot}6154\, m' - 0{\cdot}0104\, m'^2 \quad . \quad (19)$$

$$\log \bar{r}_m' = 2{\cdot}397 + 0{\cdot}183 \log B_m' \quad . \quad . \quad . \quad (20)$$

$$\log \bar{p}_m' = -2{\cdot}136 - 0{\cdot}234 \log B_m' \quad . \quad . \quad . \quad (21)$$

$$\log \bar{\pi}_m' = -1{\cdot}210 - 0{\cdot}120\, m' \quad . \quad . \quad . \quad (22)$$

By means of equations (17) these equations may be transformed so as to express the relationships corresponding to any values of α and $\bar{M}_0$.

§ 2. Comparison of Theory with Observations.

(*a*) *The Relation between* B_m *and the Photographic Magnitudes* m_p.

This relation has been examined statistically by Messrs. Chapman and Melotte (*Mem. R.A.S.*, vol. lx. part iv.) for eight

regions selected according to galactic latitude. No subdivision of the material along galactic longitudes has so far been made. It must therefore be assumed that the figures quoted represent the *average* conditions along the galactic parallels whose latitudes are stated.

The observed correlated values of m_p and log B_m are shown in table vii. of their paper, the limiting magnitudes being based on the photographic system of the North Polar Sequence published in *H.C.*, 175. For reasons stated in my previous paper, these magnitudes have been reduced to the Mount Wilson system by applying the corrections quoted in table ii., *loc. cit.*

It is proposed to show that in each region the integration of equation (12) leads to results in satisfactory agreement with the observations. This agreement is attained by accepting for all regions

$$\nu_0 = 0{\cdot}07343\,; \quad h = 0{\cdot}249,$$

while α and M_0 are varied.

Under these conditions Table I. and equations (17) will enable us to find the particular values of α and $\bar{M}_0$ which satisfy the observations in each of the eight regions.

Table II. shows the resulting values of these quantities, while in Table III. the differences between the observed and computed quantities log B_m are represented.

TABLE II.

Region.	β_0.	α.	$\bar{M}_0$.
I.	5°	0·00228	3·82
II.	15	·00254	3·86
III.	25	·00296	3·87
IV.	35	·00356	3·98
V.	45	·00429	4·09
VI.	55	·00402	4·20
VII.	65	·00435	4·22
VIII.	80	·00487	4·17

TABLE III.

Δ log B_m (O − C).

m_p.	I.	II.	III.	IV.	V.	VI.	VII.	VIII.	Mean.
7·0	+ ·01	·00	− ·07	− ·04	− ·03	− ·04	+ ·01	+ ·04	− ·01
8·0	+ ·01	+ ·01	− ·02	− ·01	+ ·01	− ·01	+ ·01	+ ·03	·00
9·0	·00	·00	− ·01	+ ·01	+ ·01	+ ·01	·00	+ ·01	·00
10·0	− ·01	·00	+ ·01	+ ·01	+ ·01	·00	− ·01	− ·02	·00
11·0	− ·01	− ·01	+ ·02	+ ·01	+ ·01	·00	− ·02	− ·03	·00
12·0	− ·01	− ·01	+ ·03	+ ·01	+ ·01	+ ·01	− ·02	− ·03	·00
13·0	− ·01	− ·01	+ ·02	+ ·01	+ ·01	+ ·01	− ·01	− ·02	·00
14·0	− ·01	·00	+ ·01	·00	·00	+ ·01	− ·01	− ·03	·00
15·0	− ·01	− ·02	·00	− ·02	− ·02	·00	·00	− ·02	− ·01
16·0	+ ·01	− ·01	·00	− ·01	− ·01	+ ·01	+ ·01	·00	·00
17·0	+ ·02	+ ·02	+ ·01	·00	·00	+ ·01	+ ·04	+ ·05	+ ·02

Though some evidence of systematic differences between the observed and computed quantities is apparent, the residuals shown in Table III. are on the whole sufficiently small to justify the acceptance of equation (11) within the present range of observations. It is, of course, probable that other powers of r may have to be taken into account in the exponent of (11), in order to represent the observed relation (B_m, m) as we proceed to still fainter stars. To draw conclusions from this equation with regard to the structure of the universe beyond the present limits of vision would doubtless be unsafe.

The material discussed by Chapman and Melotte is confined to galactic regions north of the equator. To test the above results for the southern hemisphere a similar investigation was made, utilising the material supplied by the C.P.D. catalogues. In the preceding paper "On the Magnitudes of the C.P.D." it has been shown that the relation (B_m, m) derived from C.P.D. stars situated north of $-58°$ declination is in satisfactory agreement with that shown by Chapman and Melotte's investigation for a corresponding galactic latitude.

The general conclusion derived from the southern stars may be summarised in the statement that for the same galactic latitude the modulus a is the same for northern and southern stars. A notable exceptional feature, however, is shown by the value of M_0 in the galactic region intersected by the parallel $-45°$ declination. There appears some reason to assume that in this particular region stars of all apparent magnitudes either possess relatively greater intrinsic brightnesses or are more densely crowded than in other parts of the galaxy.

The moduli a of Table II. show a distinct dependence on the galactic latitude β. From a discussion of the complete material comprising northern and southern stars, it was found that the derived figures can be satisfactorily represented by the relation

$$a_p = 0{\cdot}00183 + 0{\cdot}00307 \sin \beta \quad . \quad . \quad . (23)$$

the suffix p being used to indicate that a is derived from material based on photographic magnitudes.

The probable errors of the numerical coefficients in (23) are $\pm 0{\cdot}00010$ and $\pm 0{\cdot}00015$ respectively.

(*b*) *The Relation between* B_m *and the Visual Magnitudes* m_v.

Having shown that the relations (B_m, m_p) in different regions can be represented by integration of equation (11), considering a and M_0 as quantities depending generally on the spherical co-ordinates (λ, β), we proceed to examine from the same point of view the observed relations (B_m, m_v).

In this respect the statistical results derived by Professor Kapteyn in *Groningen Publ.*, No. 18, are of great importance. The data derived directly from the observations are found in

table S. 35 of Kapteyn's memoir, where separate results are given for three groups whose galactic latitudes lie between 0°–20°, 20°–40°, and 40°–90°. For stars brighter than the 9th magnitude the star counts are derived from catalogues of visual observations, and refer to stars whose visual magnitudes are brighter than the magnitude stated. For the fainter stars, however, the material has been chiefly collected from photographs on which stars were counted *photographically* brighter than a reference star whose *visual* magnitude is accepted as the limiting magnitude. Kapteyn's counts of the fainter stars are therefore based on a principle comparable with that adopted by Chapman and Melotte, but his limiting magnitudes differ from the photographic magnitudes by amounts equivalent to the average colour-index of stars of that brightness.

Since the fainter stars carry the principal weight in the determination of α, the subsequent calculations have been based on the assumption that Kapteyn's star counts thoughout refer to limiting magnitudes fixed by the *photographic* images of the stars, but expressed by numbers which differ from the true photographic magnitudes by amounts equivalent to the average colour-indices.

Now, the values of α_p and $\bar{M}_p$ of Table II., interpolated for the mean galactic latitudes of Kapteyn's groups, enable us to calculate from Table I. and equations (17) the *photographic* magnitudes corresponding with Kapteyn's values of log B_m. The differences between these photographic magnitudes and the visual magnitudes adopted by Kapteyn represent the colour-indices $(m_p - m_v)$.

Table IV. shows the values of α_p and $\bar{M}_p$ for the mean latitudes of Kapteyn's groups, viz. 10°, 20°, and 57°, the former being calculated from equation (23) and the latter being interpolated from Table II.

In Table V. the first two vertical columns of each group contain Kapteyn's values of m_v and log B_m. They are copied from his table S. 35, with the exception of the two faintest groups, for which it was preferred to substitute the more directly derived results of tables S. 29 and S. 30.

The third and fourth columns show m_p and log $\bar{r}_m$ derived from Table I. and equations (17) with the values of α_p and $\bar{M}_p$ of Table IV., and the fifth columns give the resulting colour-index $(m_p - m_v)$.

Table IV.

β.	α_p.	$\bar{M}_p$.
0° – 20°	0·00236	3·85
20° – 40°	0·00336	3·92
40° – 90°	0·00441	4·17

TABLE V.

$\beta=0°-20°$.						$\beta=20°-40°$.					
m_v.	log Bm.	m_p.	log $\bar{r}_m$.	(m_p-m_v).	(O − C).	m_v.	log Bm.	m_p.	log $\bar{r}_m$.	(m_p-m_v).	(O − C).
4·50	−1·56	4·86	2·169	+0·36	+·10	4·50	−1·76	4·71	2·064	+0·21	−·02
5·50	−1·04	5·88	2·268	+0·38	+·04	5·50	−1·25	5·73	2·159	+0·23	−·06
6·75	−0·43	7·12	2·379	+0·37	−·08	6·75	−0·62	7·06	2·274	+0·31	−·06
9·10	+0·79	9·83	2·594	+0·73	−·02	9·10	+0·56	9·74	2·480	+0·64	+·04
9·25	+0·87	10·03	2·609	+0·78	·00	9·25	+0·64	9·93	2·493	+0·68	+·06
9·96	+1·11	10·60	2·649	+0·64	−·21	9·96	+0·83	10·41	2·527	+0·45	−·22
10·86	+1·63	11·92	2·739	+1·06	·00	10·86	+1·30	11·64	2·611	+0·78	−·03
12·14	+2·20	13·50	2·841	+1·36	+·02	12·14	+1·90	13·35	2·719	+1·21	+·17
13·18	+2·69	15·01	2·934	+1·83	+·16	13·18	+2·28	14·53	2·790	+1·35	+·13
13·50	+2·74	15·18	2·944	+1·68	−·03	14·11	+2·59	15·57	2·851	+1·46	+·05
14·16	+3·01	16·07	2·996	+1·91	−·02	14·15	+2·57	15·51	2·847	+1·36	−·03

$\beta=40°-90°$.					
m_v.	log Bm.	m_p.	log $\bar{r}_m$.	(m_p-m_v).	(O − C).
4·50	−1·83	4·89	2·000	+0·39	·00
5·50	−1·34	5·88	2·089	+0·38	−·05
6·75	−0·72	7·20	2·200	+0·45	−·05
9·10	+0·46	9·97	2·405	+0·87	+·18
9·25	+0·50	10·07	2·412	+0·82	+·12
9·96	+0·67	10·51	2·443	+0·55	−·19
10·86	+1·18	11·89	2·534	+1·03	+·16
12·14	+1·55	12·96	2·601	+0·82	−·16
13·18	+1·96	14·24	2·679	+1·06	−·08
13·88	+2·25	15·22	2·735	+1·34	+·07
13·94	+2·25	15·22	2·735	+1·28	+·01

(*c*) *Differences between Photographic and Visual Results. Space-Absorption.*

It will be noticed that in all groups the colour-index $(m_p - m_v)$ shows a pronounced increase with apparent magnitude. The question then arises: Does this phenomenon point to the presence of terms a_ϕ or a_ψ depending on the effective wave-length?

We find that in each group the colour-indices can be represented by an equation of the form

$$m_p - m_v = (\bar{M}_p - \bar{M}_v) + 0{\cdot}00197\bar{r}_m \quad . \quad . \quad (24)$$

if to $\bar{M}_v$ the values 3·88, 3·92, and 3·98 are assigned. The residuals [Table V., equation (24)] are shown in the sixth columns under the heading (O − C).

Now, from equations (18) and (17) we find

$$m_p - \bar{M}_p = 2{\cdot}98 + \tfrac{5}{3}\log B_m + 1{\cdot}796 a_p \bar{r}_m \quad . \quad . \quad (25)$$

Hence

$$m_v - \bar{M}_v = 2{\cdot}98 + \tfrac{5}{3} \log B_m + 1{\cdot}796 a_v \bar{r}_m \quad . \quad (26)$$

where

$$a_v = a_p - 0{\cdot}00110 \quad . \quad . \quad . \quad (27)$$

Consequently, Kapteyn's visual magnitudes m_v and Chapman and Melotte's photographic magnitudes m_p are represented by equations (26) and (25) respectively, which are of the same form but differ in the numerical values of $\bar{M}$ and a.

We thus arrive at the conclusion that

The modulus a depends on the effective wave-length of the light in which the brightness of the stars has been measured.

This result is, of course, derived on the assumption that m_v and m_p are correctly determined. Since doubts have been expressed particularly with regard to the fainter visual magnitudes of Kapteyn's series, it is important to show that equation (26) is confirmed by other observations.

In a paper which appeared in the *Proceedings of the National Academy of Sciences*, 3, 217, Professor Seares has published values of log B_m from star counts made on photographic plates and referred to the limiting magnitude 17·1 (Harvard visual). The principle on which these observations were made is in accordance with that adopted by Professor Kapteyn for the fainter magnitudes. Equation (26) should therefore satisfy Seares' observations.

As an example let us take the case $\beta = 10°$, for which Professor Seares gives log $B_m = 3{\cdot}99$. With $a_p = 0{\cdot}00236$ (Table IV.) we find log $a'/a = +0{\cdot}111$, and from equation (17)

$$\log B_{m'} = 3{\cdot}66.$$

Entering with this quantity into Table I., we obtain

$$\log \bar{r}_{m'} = 3{\cdot}09,$$

and therefore

$$\log \bar{r}_m = 3{\cdot}09 + 0{\cdot}11 = 3{\cdot}20.$$

Since $\bar{M}_v = 3{\cdot}88$ and $a_v = 0{\cdot}00126$, equation (26) gives $m_v = 17{\cdot}1$, in accordance with the limiting magnitude stated by Professor Seares.

This test shows that Kapteyn's fainter magnitudes must be in good accordance with the Harvard visual system.

As to the photographic magnitudes, a close agreement between Chapman and Melotte (reduced to the Mount Wilson scale) and the Cape standard system down to the 12th magnitude has been proved in my previous paper by comparisons of star counts. Further, the photographic magnitudes fainter than the 12th were carefully standardised by direct comparisons with the fundamental stars of the North Polar Sequence.

Kapteyn's magnitudes m may therefore be considered to represent the Harvard visual system down to the faintest stars, while Chapman and Melotte's corrected magnitudes m_p represent the photographic system of Mount Wilson and the Cape. It is

unlikely that either of the two fundamental systems can be affected by errors which would give rise to such considerable changes depending on magnitude in the differences $(m_p - m_v)$ as are shown in Table V. The acceptance of this view would be tantamount to the confession of utter failure in our attempts of establishing trustworthy scales of magnitudes.

But if, on the other hand, the Harvard visual and Mount Wilson photographic systems are accepted as sufficiently correct representations of the light ratios, the colour-indices $(m_p - m_v)$ can be explained only by the fact that the modulus α depends on the *optical* conditions under which the stars are observed.

From (23) and (27) the following formulæ are obtained:

$$\left.\begin{aligned} \alpha_v &= 0{\cdot}00073 + 0{\cdot}00307 \sin\beta \\ \alpha_p &= 0{\cdot}00183 + 0{\cdot}00307 \sin\beta \end{aligned}\right\} \quad . \quad . \quad (28)$$

where α_v and α_p are the *average* moduli for the galactic parallel whose latitude is β.

Hence the modulus α can be represented by the sum of two terms, one being independent of the galactic position, and the other proportional to the *sine* of the galactic latitude. Only the first term exhibits change with change of the *optical* conditions, the factor of the second term being the same for visual and photographic observations.

This result suggests the action of an optical cause depending on the wave-length, and operating in a like fashion in all directions and in such a manner that the absolute magnitudes appear fainter in proportion to the distances at which the stars are observed.

Absorption of the star's light during its passage through space would comply with these conditions. According to Lord Rayleigh, the ratio of the loss of light due to "scattering" at wave-lengths λ_p and λ_v is $\lambda_v^4 : \lambda_p^4$. Now the effective wave-lengths λ_v and λ_p can be determined from the distribution of radiative energy in the *visual* and *photographic* portions of the spectrum.

From observations made by Professor Parkhurst, this ratio seems to be about 2·4. On the other hand, if the first terms of (28) are due to scattering, they represent the losses expressed in magnitudes at wave-lengths λ_v and λ_p suffered by a star at unit distance. We thus find the ratio 2·51, in close agreement with the ratio derived from Parkhurst's observations.

(*d*) *The Special Case* $\alpha = \alpha_f$, *Schwarzschild's Theory.*

Whatever the physical cause of the first terms of (28) may be, the fact that they depend on the effective wave-lengths is sufficient to decide that the assumption $\alpha_\phi + \alpha_\psi = 0$ is not warranted. For in this case $\alpha = \alpha_f$, and consequently the impossible result would be obtained that the frequency ν depends on the optical character of the light in which the stars are observed.

The assumption $\alpha = \alpha_f$ necessarily requires that $\alpha_\psi = \alpha_f$.

This conclusion is important because, as already pointed out, statistical theories have hitherto been based on this latter assumption, *i.e.* on a luminosity law independent of the distance.

In Schwarzschild's theory, for instance, the luminosity law has the form

$$10^{b_0 - b_1 M - b M^2} \quad . \quad . \quad . \quad . \quad (29)$$

where b_0, b_1, b_2 are quantities which may depend on the spherical co-ordinates (λ, β), but are independent of the distance ρ (see Eddington, *Stellar Movements*, p. 211).

Now for $\alpha = \alpha_f$ the luminosity law (3) takes the form

$$e^{-h^2[M-\gamma]^2},$$

where γ depends on (λ, β) but not on ρ. Hence for this particular case the luminosity law of the present theory agrees in form with that of Schwarzschild.

Further, in accordance with equations (7), (8), and (13), the frequency law is

$$\nu = \nu_0 \frac{r^2 dr}{\rho^2 d\rho} \quad . \quad . \quad . \quad . \quad (30)$$

where

$$\log \rho = \log r + \tfrac{1}{5}(\alpha_f + \alpha r) \quad . \quad . \quad . \quad (31)$$

A frequency law of this character may, within a wide range of distances, be represented by an equation of the form

$$\nu = 10^{\delta_0 - \delta_1 \log \rho - \delta_2 (\log \rho)^2} \quad . \quad . \quad . \quad (32)$$

which is the frequency law of Schwarzschild's theory.

Further, the parallaxes are proportional to $\pi = r^{-1} \cdot 10^{-\frac{1}{5}\alpha r}$, and hence by equation (22) the average parallaxes of stars of magnitude m are approximately represented by an equation of the form

$$\bar{\pi}_m = 10^{-P_0 - P_1 m} \quad . \quad . \quad . \quad . \quad (33)$$

Finally, in agreement with equation (19), B_m can be represented in the general form

$$B_m = 10^{\epsilon_0 - \epsilon_1 m - \epsilon_2 m^2} \quad . \quad . \quad . \quad . \quad (34)$$

The expressions (33) and (34) agree with corresponding equations of Schwarzschild's theory.

But the impossibility of the assumption $\alpha = \alpha_f$ is at once seen when we attempt to derive the frequency law from equations (30) and (31). Obviously, for the same distances ρ different frequencies ν would be obtained, according to whether α is determined from the visual or photographic magnitudes, *i.e.* from the first or second equation (28). But the *total* number of stars in unit volume cannot possibly depend on the conditions of luminosity.

Hence if the difference $(m_p - m_v)$ found in section (*c*) is due to a *vera causa*, the frequency law derived from Schwarzschild's theory must be erroneous.

It is highly desirable, therefore, that in future statistical work the derivation of the modulus α from observations based on visual

and photographic magnitudes should receive careful attention. The establishment of correct magnitude scales for at least two different effective wave-lengths on the one hand, and of star counts for a great number of selected areas on the other, would supply the necessary material. If the discussion of the relations (B_m, m_v) and (B_m, m_p) should lead to the result $\alpha_v = \alpha_p$, then the assumption $\alpha = \alpha_f$ would be justified, and the frequency law derived from Schwarzschild's theory might be accepted. If, however, future statistics support the conclusion to which the present observations seem to point, viz. that α_v and α_p are *different* quantities, then the assumption of a uniform luminosity law and of the absence of space-absorption cannot be admitted.

Until this alternative is finally settled, views with regard to the structure of the universe, which so far have always been based on the assumption of a uniform luminosity law and on the absence of space-absorption, must be considered extremely doubtful.

We shall now briefly discuss the evidence regarding the presence or absence of coefficients α_ϕ and α_ψ derived from the proper motions.

(*e*) *Comparison of the Distances derived from the Proper Motions with the Parameters* r.

The parallactic motions enable us to find the average parallaxes of stars of known apparent magnitude. Investigations in this direction have been chiefly made by Professor Kapteyn for stars brighter than the 9th magnitude, and recently (*M.N.*, 77, 7) by Sir Frank Dyson and Mr. Thackeray, including stars down to magnitude 10·3. The corresponding values of m (Harvard visual) and the observed mean parallaxes—with the addition of a result obtained by Professor Comstock for stars fainter than 10^m—are shown in the first two vertical columns of Table VI. (The individual results given in Dyson and Thackeray's paper are here combined into groups comprised within the magnitudes: bright—6·9, 7·0–7·9, 8·0–8·9, 9·0–9·9, 10>, weights having been assigned in proportion to the number of stars examined.)

The observed parallaxes are to be compared with the theoretical quantities $\bar{p}_m$ and $\bar{\pi}_m$ derived from Table I. for the same magnitudes. The values of $\bar{M}_0$ and α_v which satisfy the relation between B_m and m_v (Harvard visual) for stars taken over the entire sphere are

$$\bar{M}_0 = 3{\cdot}90; \quad \alpha_v = 0{\cdot}00195.$$

Since Table I. is calculated with $\bar{M}_0' = 3{\cdot}95$, $\alpha' = 0{\cdot}00305$, we obtain from equation (17)

$$m' = m - 0{\cdot}92.$$

With the argument m' Table I. was entered and the corresponding quantities $\log \bar{p}_m'$ and $\log \bar{\pi}_m'$ were extracted. Finally, from these, in accordance with the first equation (17), $\log(\alpha'/\alpha) = 0{\cdot}194$ was deducted, and thus $\log \bar{p}_m$ and $\log \bar{\pi}_m$ were obtained, whose natural numbers are shown in columns 3 and 4 of Table VI.

TABLE VI.

m_v (Harv.).	Observed Parallaxes.	$\bar{p}_m$.	$\bar{\pi}_m$.	Authority.
	"	"		
2·56	0·0362	0·0352	0·0341	Kapteyn.
3·97	·0194	·0213	·0203	,,
4·90	·0139	·0156	·0147	,,
5·84	·0122	·0116	·0107	,,
6·17	·0100	·0105	·0097	Dyson and Thackeray.
6·76	·0085	·0088	·0080	Kapteyn.
7·37	·0079	·0075	·0067	Dyson and Thackeray.
8·39	·0062	·0057	·0049	,, ,,
8·72	·0060	·0053	·0045	Kapteyn.
9·26	·0047	·0046	·0038	Dyson and Thackeray.
10·22	·0032	·0037	·0029	,, ,,
10·5	·0046	·0035	·0027	Comstock.

Comparison between the observed parallaxes of column 2 and the theoretical quantities $\bar{p}_m$ and $\bar{\pi}_m$ shows that the true distances are somewhat better represented by the assumption $\rho = r$ than by $\rho = r10^{\frac{1}{5}ar}$, the latter assumption indicating a spread of stars in distance in excess of that shown by the parallaxes obtained from the parallactic motions.

Hence the assumption $a = a_f$ (Schwarzschild's and Kapteyn's theories) appears to make the dimensions of the universe somewhat too great.

Considered by itself, the weight of this conclusion may seem small, on account of the minuteness of the differences between $\bar{p}_m$ and $\bar{\pi}_m$, and the still considerable uncertainties in the values of the observed parallaxes.

But it is remarkable that the same conclusion has been reached by Eddington from a different point of view. From a discussion of the *motus peculiares* of the stars in Boss's General Catalogue, Eddington derived the law of distribution in distance of these stars. Comparing with Kapteyn's theory, he found that "for stars of a given magnitude the spread in distance must be less than that deduced from Kapteyn's formulæ." Since the distances of Kapteyn's theory agree with those derived from the preceding theory on the assumption $a = a_f$, Eddington's result therefore supports the conclusion derived from Table VI.

§ 3. SUMMARY AND CONCLUSIONS.

1. In common with other statistical investigations, the present theory is based on a luminosity law of the form shown in equation (3). Statistical theories were so far based on the hypothesis that the luminosity law (3) is the same in all parts of space, and that no appreciable space-absorption exists (see Eddington, *Stellar*

Movements, p. 202). In the present paper, however, a more general view is taken by considering $\bar{M}$ as an unknown function of the co-ordinates (ρ, λ, β), while accepting the universal constancy of the modulus h, and also by admitting the existence of an influence of the propagating medium on the radiative energies. These assumptions are introduced tentatively, the decision whether they are acceptable to be tested by observations. Further, defining ν, the frequency, *i.e.* the total number of stars per unit volume, as an unknown function of (ρ, λ, β), the fundamental equation of stellar statistics in its most general form is derived in equation (9).

2. Comparison with observation showed that, within the present range of observational data, the sum of the functions $(f + \phi + \psi)$ can be represented as a *linear* function of a parameter r. The relations between B_m and m, observed in any part of the sky, can be satisfied by integration of the simplified equation (11), assigning to ν_0 and h *fixed* numerical values and considering α and $\bar{M}_0$ as functions of (λ, β). These latter terms are represented by the sums

$$\alpha = \alpha_f + \alpha_\phi + \alpha_\psi\,; \quad \bar{M}_0 = a_f + a_\phi + a_\psi.$$

The coefficients α_f and a_f define the frequency law, the coefficients α_ϕ and a_ϕ define the luminosity law, and the coefficients α_ψ, a_ψ indicate the effect of the propagating medium on the radiative energies.

3. In order to form definite conclusions regarding these laws, a separation of the component parts of the α and a coefficients is essential. For this purpose equation (11) was compared with observed relations (B_m, m) derived from photographic and visual magnitudes. Thus the quantities α_p, $\bar{M}_p$ and α_v, $\bar{M}_v$ for different galactic latitudes were obtained (§ 2, *a*, *b*).

4. In the absence of coefficients of the type α_ϕ and α_ψ, the moduli α_p and α_v must be identical. The observed data, however, show distinctly different values. Provided that these data are reliable, the difference $(\alpha_p - \alpha_v)$ proves the presence of a coefficient $(\alpha_\phi + \alpha_\psi)$ of considerable magnitude, part of which may probably be due to selective space-absorption (scattering) (§ 2, *c*).

5. This result shows that the assumption $\alpha = a_f$, which leads to the statistical equations of Schwarzschild's theory, is not justified (§ 2, *d*).

6. Comparisons between the distances obtained from the proper motions on the one hand, and star counts on the other, lead to the conclusion that distances derived from the assumption $\alpha = a_f$ are too great. The results of the two methods are brought into better agreement when the existence of coefficients of the types α_ϕ and α_ψ is accepted.

7. Though a separation of the coefficients α_f, a_f by the method indicated in this paper is theoretically possible, more refined and extended observational data are needed for this purpose. An insight into the real structure of the universe is for this reason not yet possible. But the evidence of the observations at present

available points distinctly to the conclusion that the falling off in the number of stars with increasing magnitude and distance is, at least partly, an *optical* phenomenon.

8. For a complete separation of the frequency and luminosity laws in any part of the sky, three empirical relations must be established by observation, viz.—

(*a*) The relation between B_m and m_v ;

(*b*) The relation between B_m and m_p, where m_v and m_p signify magnitudes determined for starlight of *different* effective wave-lengths ;

(*c*) The relation between m_v (or m_p) and the average parallaxes determined from the parallactic motions.

From (*a*) and (*b*) the quantities

$$\bar{M}_v = a_f + [a_p + a_\psi]_v ; \quad \bar{M}_p = a_f + [a_\phi + a_\psi]_p,$$
$$a_v = a_f + [a_\phi + a_\psi]_v ; \quad a_p = a_f + [a_\phi + a_\psi]_p$$

can be determined. Also the relation between m_v (or m_p) and the theoretical parallaxes $\bar{p}_m$ can be established.

Comparing the latter with the true parallaxes obtained from (*c*), the coefficients a_f and a_f, which define the frequency law only, are obtained.

The residual coefficients $(a_\phi + a_\psi)$ and $(a_\phi + a_\psi)$ may be due to either intrinsic radiative properties of the stars conditioned by their sizes and temperatures, or to the action of space-absorption.

Such an investigation, extended over a great number of selected areas in all parts of the sky, will eventually lead to the special form of the law expressing ν as a function of the space co-ordinates (ρ, λ, β).

Unfortunately reliable parallaxes from the parallactic motions for such faint stars as are required in this investigation cannot be expected for many years to come. There is no reason, however, why the observations (*a*) and (*b*) should not be undertaken at once on a comprehensive scale. The valuable researches of Chapman and Melotte form already an important step in this direction. It remains, on the one hand, to extend their work over the entire sphere, and on the other to undertake a fresh series of photographic observations based on the same plan, in which the effective wave-length is changed by the use of a coloured light filter in front of the plate. If sufficient care is taken to determine the magnitudes on the correct photometric scale, the important question whether α depends on the wave-length could be answered.

In conclusion, attention is drawn to a phenomenon recently pointed out by Professor Turner, which may be stated as follows :— If two limiting magnitudes m_1 and m_2 are selected and the corresponding star numbers B_1 and B_2 are counted in different regions of the sky, the ratio B_2/B_1 is found to be a function of the position. It can be readily seen from Table I. and equations (17) that variations of α and $\bar{M}_0$ would account for this phenomenon.

From equations (17), (18), and (20) we find

$$[m_2 - m_1] - \tfrac{5}{3}[\log B_2 - \log B_1] = A[B_2^{0\cdot183} - B_1^{0\cdot183}],$$

where

$$A = 1\cdot366(\alpha/\alpha')^{0\cdot549}; \quad \alpha' = 0\cdot00305.$$

Also

$$\bar{M}_0 = m_1 - 2\cdot98 - \tfrac{5}{3}\log B_1 - AB_1^{0\cdot183} = m_2 - 2\cdot98 - \tfrac{5}{3}\log B_2 - AB_2^{0\cdot183}.$$

Hence from the given data $[m_1, m_2, B_1, B_2]$ α and $\bar{M}_0$ can be derived in any part of the sky, and if these data are known for visual as well as photographic magnitudes, the moduli α_v and α_p can be determined separately.

As an example let us take Chapman and Melotte's observations in Zones I. and VIII. for $m_1 = 8$ and $m_2 = 16$, which correspond to 8·09 and 16·25 on the Mount Wilson scale (see table ii. of my previous paper). From table vii. on p. 157 of their paper we extract

m.	I. log B.	VIII. log B.
8·09	0·065	−0·330
16·25	3·095	2·470

which gives

for Zone I.: $2\cdot65A = 3\cdot11$; $\alpha = 0\cdot00231$, $\bar{M}_0 = 3\cdot79$,

„ „ VIII.: $1\cdot96A = 3\cdot49$; $\alpha = 0\cdot00494$, $\bar{M}_0 = 4\cdot12$,

in good agreement with Table II. of the present paper.

This simple method of finding α and $\bar{M}_0$ may prove useful in future statistical investigations.

Royal Observatory, Cape of Good Hope:
1917 *August.*